Oxford Physics Aptitude Test (PAT) 2006 Solutions

Physics

1) velocity × time = displacement Ans: C

2) $V = 3\times4\times5\times10^{-6} = 6\times10^{-5}\,m^3$

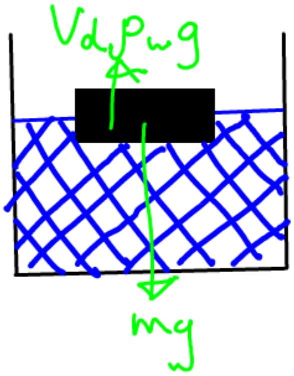
$m = \rho V = 8570 \times 6\times10^{-5} = 0.5142\,kg$

$p = \dfrac{F}{A} = \dfrac{mg}{\underbrace{3\times4\times10^{-4}}_{\text{smallest area}}} = \dfrac{5.142}{1.2\times10^{-3}} = 4285\,Pa$

Ans: A

3) At D, potential energy is higher compared to others so kinetic energy is lower. Ans: A

4)



Volume displaced $= V_d$ Density of water $= \rho_w$
mass of boat $= m$

$mg = V_d \rho_w g$

$V_d = \dfrac{m}{\rho_w}$

Add an extra mass to the boat, $\delta m = \rho_w \delta V$

$\Rightarrow V_d' = \dfrac{m+\delta m}{\rho_w} = \dfrac{m}{\rho_w} + \dfrac{\rho_w \delta V}{\rho_w} = V_d + \delta V$

But δV was taken from the water

$\therefore V_d'' = V_d + \delta V - \delta V = V_d$ Ans: C

5) $eV = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$

$\lambda = \dfrac{h}{\sqrt{2meV}}$ Ans: B

6) $s = \left(\frac{u+v}{2}\right)t$

$t = \frac{2d}{20+0}$

$= 0.1d$

$s' = \left(\frac{0+20}{2}\right) 0.2d$

$= 2d$

Ans: D

7) D: 1, 4, 9, 16, 25 ... n^2

Y: 1, 8, 27, 64, 125 ... n^3

$1\sqrt{1}$ $4\sqrt{4}$ $9\sqrt{9}$ $16\sqrt{16}$ $25\sqrt{25}$ $n^2\sqrt{n^2}$

Ans: C

8) Reading on scale, $R = \frac{W}{8.8} = \frac{mg}{8.8}$

On Mars, $R = 93 = \frac{3.8m}{8.8}$

$m = 215\text{ kg}$ Ans: C

9) Mass is constant since there is no loss of matter

Given that volume decreases, density must increase

$m = V\rho$ Ans: B

10) ↓+ve $s = ?$ $u = 11\text{ ms}^{-1}$ $v = x$ $a = 10\text{ ms}^{-1}$ $t = 7\text{ s}$

$s = ut + \frac{1}{2}at^2 = 11(7) + \frac{1}{2}(10)7^2 = 322\text{ m}$ Ans: C

11) Brightness is determined by power, $P = \frac{V^2}{R}$

a) Half the voltage ⇒ dimmer

b) Same voltage ⇒ normal

c) Same voltage (d is off) ⇒ normal

d) No current through d ⇒ off

e) Half the voltage ⇒ dimmer

f) Same voltage ⇒ normal

g) Double the voltage ⇒ brighter

h) Same voltage ⇒ normal

12) a ⇒ $r + g = 35$ (1)

b ⇒ $2g + b = 70$ (2)

c ⇒ $r + b = 2g$ (3)

d ⇒ $r^3\rho + g^3\rho + b^3\rho = 20000$ (4)

(2) − (3): $2g + b - r - b = 70 - 2g$

$4g - r = 70$ (5)

(1) + (5): $5g = 105$

$g = 21\text{cm} \Rightarrow r = 14\text{cm} \Rightarrow b = 28\text{cm}$

In (4): $\rho = \dfrac{20000}{14^3 + 21^3 + 28^3} = 0.589\text{g/cm}^3$ (cubes)

Density of liquid $= 2\rho = 1178\ \text{kgm}^{-3}$

13) a) Rocket started slowing down

b) Just before X. Fuel is used up so mass decreases.

c) Rocket decelerates uniformly

d) Find the area under the graph. The straight line should be extended until it reaches the x-axis.

14) a) $KE = Pt = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2Pt}{m}}$$

b) $a = \frac{dv}{dt} = \frac{1}{2}\sqrt{\frac{2P}{m}} \cdot t^{-1/2} = \sqrt{\frac{P}{2mt}}$

$$d = \int_0^t v\,dt = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}\,dt = \sqrt{\frac{2P}{m}}\left[\frac{2}{3}t^{3/2}\right]_0^t$$

$$= \frac{2}{3}\sqrt{\frac{2Pt^3}{m}}$$

c) As $t \to \infty$, $v = \sqrt{\frac{2Pt}{m}} \to \infty$

This is not reasonable, as infinite velocity is not possible

d) As $t \to \infty$, $a = \sqrt{\frac{P}{2mt}} \to 0$

Reasonable since acceleration cannot go on forever

As $t \to 0$, $a \to \infty$

Reasonable, showing rapid initial acceleration

e) $Pt = mgh$

$h = \frac{Pt}{mg} \Rightarrow v = \frac{h}{t} = \frac{P}{mg}$

f) $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{P^2}{m^2g^2} = \frac{P^2}{2mg^2}$

$$\frac{KE}{GPE} = \frac{P^2}{2mg^2} \cdot \frac{1}{Pt} = \frac{P}{2mg^2t}$$

This is small for low P and long t.

Maths

1) i) $2007^2 - 2006^2 = (2007 - 2006)(2007 + 2006)$

$= 4013$

ii) $1.001^6 - 1.001^5 = (1 + 0.001)^6 - (1 + 0.001)^5$

$= 1 + 0.006 - 1 - 0.005 = 1 \times 10^{-3}$

2) $m = -\frac{10}{1} = -10$

3) i) $\log_e(e^{3x}) = 6$

$3x = 6$

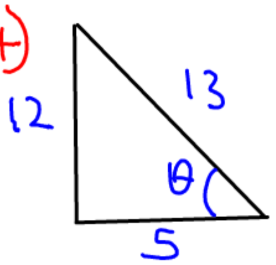
$x = 2$

ii) $\log_3 x^2 = 2$

$x^2 = 3^2$

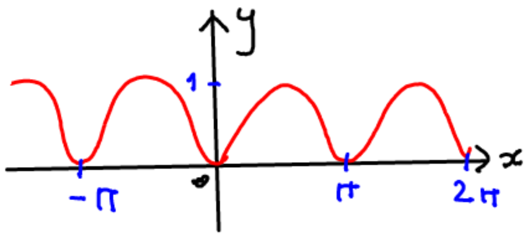
$x = \pm 3$

4)



$13\sin\left[\tan^{-1}\left(\frac{12}{5}\right)\right] = 13\sin\theta = 13 \times \frac{12}{13} = 12$

5) i)



ii) $y = \frac{1}{x^2 - 1}$

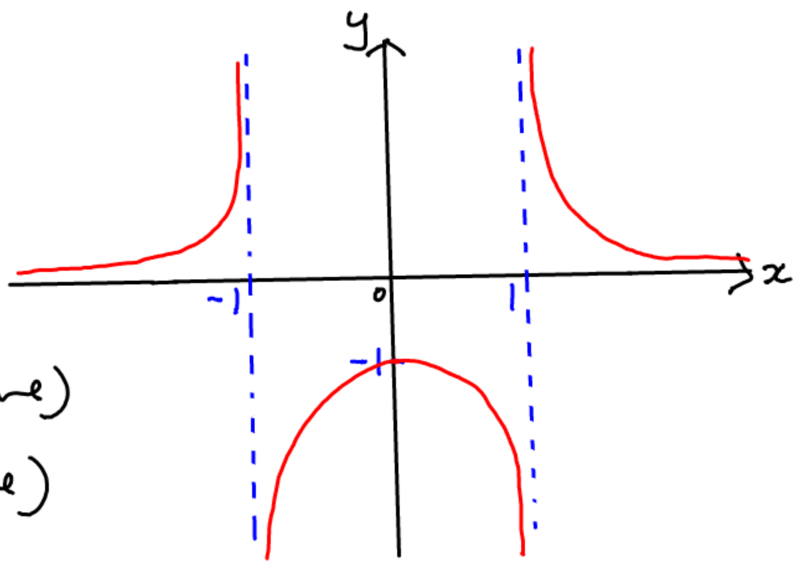
when $x = 0$, $y = -1$

when $y = 0$, no solution

$x = \pm 1$ are asymptotes

As $x \to \infty$, $y \to 0$ (+ve)

As $x \to -\infty$, $y \to 0$ (+ve)



6) $r_A = r_B + 1$ ① $A_A = A_B + 2\pi$

$\pi r_A^2 = \pi r_B^2 + 2\pi$

$r_A^2 = r_B^2 + 2$ ②

① in ② : $(r_B + 1)^2 = r_B^2 + 2$

$r_B^2 + 2r_B + 1 = r_B^2 + 2$

$r_B = \frac{1}{2} \Rightarrow r_A = \frac{3}{2}$

7) i) $P(666) = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$

ii) $P(111) + P(222) + \dots + P(666) = 6 \times \frac{1}{216} = \frac{1}{36}$

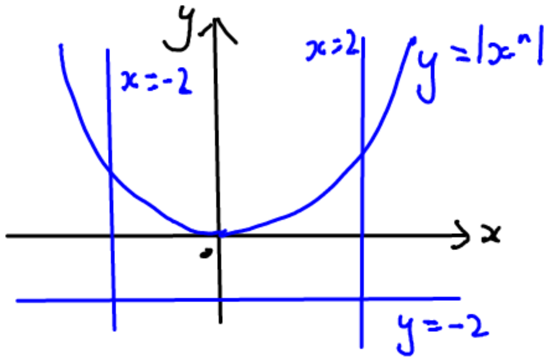
iii) $P(6'6'6) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

8) $\frac{dV}{dt} = 1 \text{cm}^3\text{s}^{-1}$ $V = \frac{4}{3}\pi r^3$ when $S = 100$,

$\frac{dV}{dr} = 4\pi r^2$ $\frac{dV}{dr} = 4\pi r^2 = 100$

$\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt} = \frac{1}{100} \times 1 = 0.01 \text{cms}^{-1}$

9)



$A = (4 \times 2) + 2\int_0^2 x^n \, dx = 8 + 2\left[\frac{x^{n+1}}{n+1}\right]_0^2$

$= 8 + \frac{2 \times 2^{n+1}}{n+1} = 8 + \frac{2^{n+2}}{n+1}$

10) i) G.P. with $a = 1$, $r = e^y$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-e^y}$

ii) $\log_2 1 + \log_2 2 + \log_2 4 + \dots + \log_2 2^n$

$= 0 + 1 + 2 + \dots + n$

A.P. with $a = 1$, $d = 1$, $\ell = n$

$S_n = \frac{n}{2}(a + \ell) = \frac{n(n+1)}{2}$

11) $y = 5 + 24x - 9x^2 - 2x^3$

$\frac{dy}{dx} = 24 - 18x - 6x^2 = 0$

$x^2 + 3 - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4 \text{ or } 1$

$\frac{d^2y}{dx^2} = -12x - 18$

When $x = -4$, $y = 5 + 24(-4) - 9(16) + 2(64)$

$= -107$

$\frac{d^2y}{dx^2} = -12(-4) - 18 = 30 > 0$

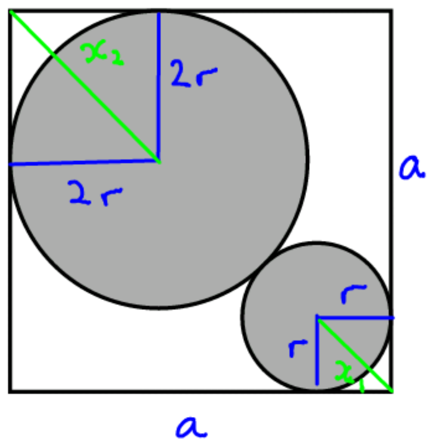
$\therefore$ $(-4, -107)$ is a min. pt.

When $x = 1$, $y = 5 + 24 - 9 - 2 = 18$

$\frac{d^2y}{dx^2} = -12(1) - 18 = -30 < 0$

$\therefore$ $(1, 18)$ is a max. pt.

12)



$x_1^2 = r^2 + r^2$ $\qquad$ $x_2^2 = (2r)^2 + (2r)^2$

$x_1 = \sqrt{2}\, r$ $\qquad$ $x_2 = 2\sqrt{2}\, r$

Diagonal of square:

$$D = x_1 + x_2 + r + 2r = (3\sqrt{2} + 3) r$$

$$D^2 = a^2 + a^2 = 2a^2$$

Area of square, $A_s = a^2 = \dfrac{D^2}{2} = \dfrac{(3\sqrt{2}+3)^2 r^2}{2}$

Area of circles, $A_c = \pi r^2 + \pi (2r)^2 = 5\pi r^2$

$\Rightarrow$ Fraction covered $= \dfrac{5\pi r^2}{\frac{(3\sqrt{2}+3)^2 r^2}{2}} = \dfrac{10\pi}{27 + 18\sqrt{2}}$