

# Year 12 Problem-Solving Course

## Section 4a: Combinatorics - counting

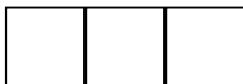
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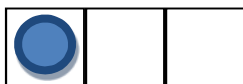
### Exercise

If you are already familiar with  ${}^nC_r$  - also written as  $\binom{n}{r}$  - from school then feel free to skip this question. It is part of the Year 12 syllabus but you might not have looked at it yet, in which case it is worth having a go now.

1. Here is a row of three boxes



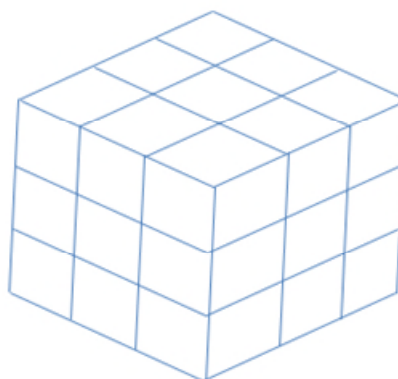
How many different pictures are there of the three boxes, in which one of them contains a spot? One such picture is like this:



How many different pictures are there of the three boxes, in which two of them contains a spot? One such picture is like this:



2. This cube made up of 27 small cubes is painted red. How many of the cubes have the following number of faces painted red?



- i) 3 ii) 2 iii) 1 iv) 0

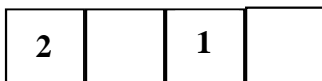
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*Again, if you are very confident about combinations and permutations from school then you may wish to skip this question.*

3. Here is a row of four boxes



How many different pictures are there of the four boxes, in which the numbers 1 and 2 have been inserted into two of them? One such picture is like this:



How many different pictures are there of the four boxes, in which two of them contains a spot? One such picture is like this:



*The following is a ‘clever’ question and it is good to hear the answer, but it is not an essential question, so you can choose to skip it or ask for help as soon as you want.*

4. There are 43 girls and 37 boys in the sixth form. There are 45 students in year 12 and 35 in year 13.  
How many more girls are there in year 12 than there are boys in year 13?
5. **UKMT:** In the highest scoring match of the World Cup, Hongeria beat Alduras 19 goals to 9. If Alduras were never in the lead, how many possible scores could there have been at half time? Thinking just in terms of the order in which goals are scored (i.e. which team), in how many ways could the final score end up being Hongeria 19 Alduras 9, if we no longer assume that Alduras were never in the lead? (For example, Alduras score the first nine goals and then Hongeria score the last nineteen goals is one possibility.) There is no need to simplify this answer!
6. **LMS, Fun Maths Roadshow:** Ian has eight identical blue socks and six identical red socks all mixed up in a drawer. What is the minimum number of socks that Ian has to take out to be sure that he has taken out at least two of the same colour? What is the minimum number for Ian to be sure that he has a sock of each colour? For both questions, imagine that Ian is drawing socks at random and can’t just look in the drawer!

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*The following question can be skipped if you have already seen this in school.*

7. [Calculation of  $\binom{n}{r}$  or  ${}^nC_r$  can often be attempted without a calculator, by means of the formula  $\binom{n}{r} = \frac{n \times (n-1) \times (n-2) \times \dots \times (n-r+1)}{r!} = \frac{n!}{(n-r)!r!}$ . It is also useful to remember that  $\binom{n}{r} = \binom{n}{n-r}$ . **Please just ask a helper to explain this, and tell you why it is useful, if you have not seen this before.]**

- a) Evaluate, without a calculator, the following

$$\binom{8}{1}; \binom{8}{2}; \binom{8}{3}; \binom{8}{4}; \binom{20}{5}.$$

- b) Write down the following  $\binom{8}{7}; \binom{8}{6}; \binom{8}{5}; \binom{20}{15}$ .

***You may have seen questions similar to the below as part of the Statistics 1 module. Skip if you feel you have already seen much of this before.***

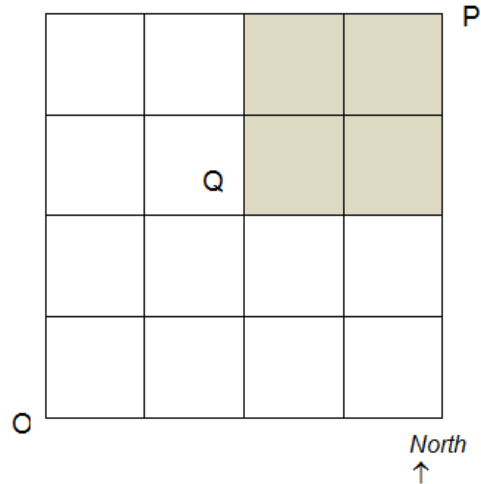
8. \*Martha Washington, wife of George Washington, had all her table linen at Mount Vernon numbered, so that it could be rotated in use.
- a) In how many ways could Mrs W(ashington) choose 14 damask table napkins on a day when she has 18 in her linen cupboard?
- b) On the same day, Mrs W also has 20 plain linen napkins in her cupboard. In how many ways could she choose 32 napkins, of any sort.
- c) If, on the same day, Mrs W had wanted to choose 14 damask and 18 plain table napkins, in how many ways could she have done this?
- d) Let us suppose that Mrs W is reviewing her linen cupboard and that she has 30 damask and 36 plain table napkins altogether. She arranges them in a long line, so that her housekeeper can check which ones need mending. How many different arrangements would there be, if Mrs W took no account of the code numbers on the napkins (so all she cared about was whether a napkin is damask or plain – wouldn't you?! – and not exactly which napkin it is!)? Give your answer as a binomial coefficient, i.e. in the form  ${}^nC_r$ .

*[\*This is attested by the Mount Vernon archives. The large numbers are made up but Mrs W did a huge amount of official entertaining. Also, well-off women in the C18 had large stores of household linen because laundering it was so laborious.]*

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*This is a great question but an infuriating question if you have not seen a trick for this before. Rather than try to find every single path below, try to think of a strategy as to how you can make the problem a whole lot easier... and don't be afraid to ask for help.*

9. **UKMT:** The diagram shows part of a grid system of city streets. For ease of reference, suppose that the intersections are given coordinates in the usual way, so that O is  $(0,0)$ , P is  $(4,4)$  and Q is  $(2,2)$ . Yolanda always navigates the grid by moving east or north.



- In how many ways, along streets in the grid, can Yolanda walk from O to P?
- How many of Yolanda's walks in part (a) pass through Q?
- The streets *inside* the shaded region are closed because of demolition work. How many ways are there now for Yolanda to get from O to P?