Year 12 Problem-Solving Course

Section 4b: Combinatorics – placement and more counting

*Questions marked **UKMT** are reproduced with permission of the United Kingdom Mathematics Trust: http://www.ukmt.org.uk/*

*Questions marked **NRICH** are reproduced with permission of NRICH: http://nrich.maths.org/*

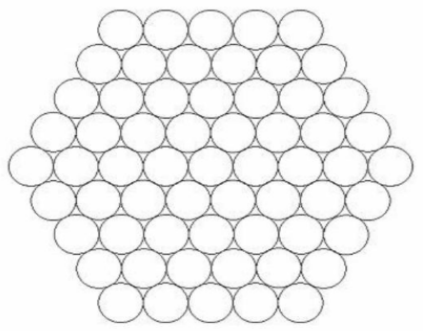
*Question marked **LMS, Fun Maths Roadshow** are reproduced with permission of the Liverpool Mathematics Society: http://www.livmathssoc.org.uk/*

Exercise

1. **NRICH:** Imagine a three dimensional version of noughts and crosses played in a 3 × 3 × 3 cell cube. How many different winning lines are there?

2. **NRICH:** Cables can be made stronger by compacting them together in a hexagonal formation.

   Here is a 'size 5' cable made up of 61 strands:

   How many strands are needed for a 'size 10' cable?

   How many for a 'size *n*' cable?

3. **NRICH:** Lee was writing all the counting numbers from 1 to 1000. She stopped for a rest after writing one thousand digits. What was the last digit she wrote?

4. **NRICH:** Arrange the numbers 0 – 20 (inclusive) into seven separate groups each of three numbers so that when the numbers in each are added together, they make seven consecutive numbers? (NB. There is not a unique solution).

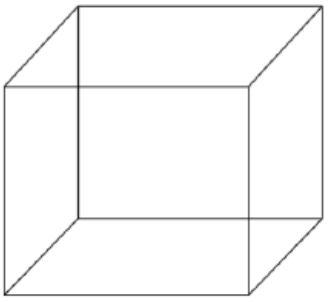
5. (a) Sally has a left sock, a right sock, a left shoe and a right shoe. In how many ways can Sally put on her socks and shoes if each foot's sock must go on before each foot's shoe?

   (b) Spiffy the spider has 8 legs, each of which is a different colour. Spiffy has a sock and a shoe for each of his 8 legs (so he has one sock and one shoe in each of the 8 colours). In how many ways can Spiffy put on his socks and shoes, if each sock and shoe must go on the leg with matching colour, and the sock must go on before the shoe on each leg? (Do not attempt to simplify your answer!)

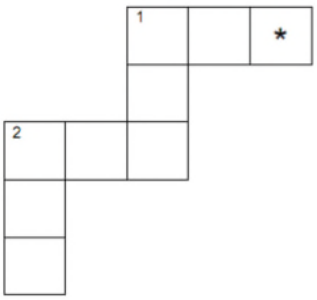
Year 12 Problem-Solving Course

6. A party of wizards is attending the Hogwarts old pupils' reunion. Of the 40, 28 have brought a cloak, 32 have brought a wand and 16 have brought an owl. Although 2 wizards have brought none of these items, there are 4 who have brought all 3. Use the inclusion-exclusion principle to find the number of wizards who have brought exactly 2 of the listed items.

7. **UKMT:** Eight points are placed at the vertices of a cube. In total, how many triangles can be formed by joining three of these points?



8. **LMS, Fun Maths Roadshow:** Each of the digits 1 to 9 is used exactly once to solve this cross number. Which digit should be placed in the square marked *



1 Across: a triangular number.

2 Across: both a square and a cube.

1 Down: a square.

2 Down: a square.

Do this if you like puzzles!

9. **LMS, Fun Maths Roadshow:** You have 15 distinct triplet-pairs of the numbers 1, 2, 3, 4, 5, 6, to be placed on the diagram so that, in any row or column, the same pair does not appear twice. A start has been made to help you.

(12)(34)(56)

(13)( )( ) (15)( )( )

(14)( )( ) (16)( )( ) (12)( )( )

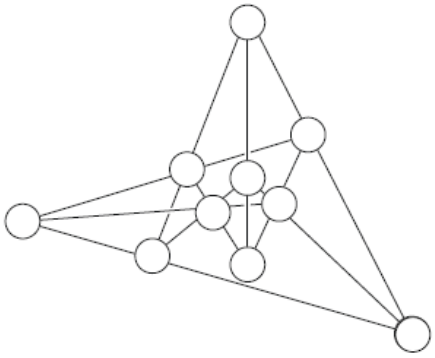
(15)( )( ) (14)( )( ) (16)( )( ) (13)( )( )

(16)( )( ) (13)(25)(46) (14)( )( ) (15)( )( ) (12)( )( )

Year 12 Problem-Solving Course

Do this if you like puzzles!

10. **LMS, Fun Maths Roadshow:** In this figure, there are ten points in space lying on ten lines. There are three of the points on each line and three of the lines through each point. Place the ten numbers



2, 3, 4, 5, 6, 7, 9, 10, 13, 15

at these points in such a way that, on each line, one of the three numbers on it is the sum of the other two.

11. **LMS, Fun Maths Roadshow:** A positive integer is written on each of the six faces of this cube, the numbers all being different. For each pair of numbers on opposite faces, one is either twice or three times the other. The sum of the six numbers is less than 160. What are the three hidden numbers?