Year 12 Problem-Solving Course

Section 3: Number - prime factorisations and irrationals

Mathematics Admissions Test Questions

Question A from the 2007 Paper

A. Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

(a) $r + s \leqslant 0$,
(b) $s \leqslant 0$,
(c) $r \leqslant 0$,
(d) $r \geqslant s$.

For solution see:
https://www.maths.ox.ac.uk/system/files/attachments/websolutions07.pdf

Question B from the 2012 Paper

B. Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers. Then N will definitely be a square number whenever

(a) k is even;
(b) $k + n$ is odd;
(c) k is odd but $m + n$ is even;
(d) $k + n$ is even.

For solution see:
https://www.maths.ox.ac.uk/system/files/attachments/websolutions12.pdf

Question 2 from the 2010 Paper

2. For **ALL APPLICANTS.**

Suppose that a, b, c are *integers* such that

$$a\sqrt{2} + b = c\sqrt{3}.$$

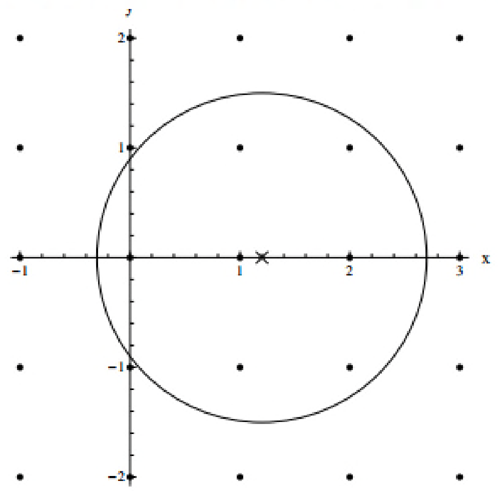
(i) By squaring both sides of the equation, show that $a = b = c = 0$.

[You may assume that $\sqrt{2}, \sqrt{3}$ and $\sqrt{2/3}$ are all irrational numbers. An irrational number is one which cannot be written in the form p/q where p and q are integers.]

(ii) Suppose now that m, n, M, N are *integers* such that the distance from the point (m, n) to $(\sqrt{2}, \sqrt{3})$ equals the distance from (M, N) to $(\sqrt{2}, \sqrt{3})$.

Show that $m = M$ and $n = N$.

Given real numbers a, b and a positive number r, let $N(a, b, r)$ be the number of integer pairs x, y such that the distance between the points (x, y) and (a, b) is less than or equal to r. For example, we see that $N(1.2, 0, 1.5) = 7$ in the diagram below.



(iii) Explain why $N(0.5, 0.5, r)$ is a multiple of 4 for any value of r.

(iv) Let k be any positive integer. Explain why there is a positive number r such that

$$N\left(\sqrt{2}, \sqrt{3}, r\right) = k.$$

For solution see
https://www.maths.ox.ac.uk/system/files/attachments/websolutions10.pdf