The Further Mathematics Support Programme

Magnificent Frigatebirds

Biologists and other scientists may collect data to try to establish whether there is a relationship between two different variables. The data they collect is known as bivariate data and is treated as coordinate pairs. The branch of statistics known as correlation allows us to calculate measures which tell us to what degree the variables are related. This example concerns data collected on ‘Magnificient Frigatebirds’.

Magnificent Frigatebirds live along tropical coastlines in southern Florida and the Gulf Coast. During the breeding season, males inflate their large, red throat sacs. They stretch their wings out and throw back their heads to make their sac look bigger.

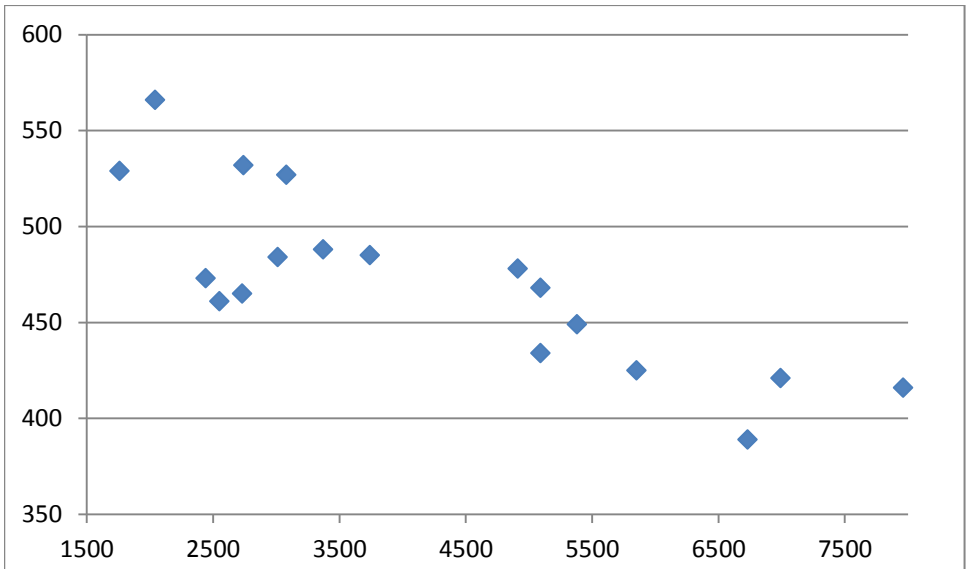
As females try to find a mate, the males twist and bend to make their throat sac look as large as possible and also make a loud, drumming noise. The birds form mating pairs each season but do not maintain the same mate from season to season.

Madsen et al. (2004) wanted to know whether females, choosing mates based on their pouch size, could use the pitch of the drumming sound as an indicator of pouch size. The authors estimated the volume of the pouch and the fundamental frequency of the drumming sound in 18 males.

The data below summarises their results:

Volume (cm^3)	Frequency (Hz)
1760	529
2040	566
2440	473
2550	461
2730	465
2740	532
3010	484
3080	527
3370	488
3740	485
4910	478
5090	434
5090	468
5380	449
5850	425
6730	389
6990	421
7960	416

To get an idea of whether there is any relationship between size of throat sac (volume) and frequency, we can firstly plot a scattergraph, plotting volume on the x-axis and frequency on the y-axis:



The scattergraph leads us to believe that there is a good negative correlation, as there is a definite downward trend, and we can work out a correlation coefficient to confirm this.

The **product moment correlation coefficient** (r) measures the correlation between the 2 variables. It takes values between 1 and -1.

1 corresponds to perfect positive correlation and -1 to perfect negative correlation.

A value of 0 means there is no correlation at all.

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$$

Where $S_{xy} = \sum xy - \frac{\sum x \sum y}{n}$ $S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n}$ $S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$

There are other ways of defining *r* using slightly different quantities, but these are the ones that are often used in A- level Mathematics and they are calculator or spreadsheet friendly.

$\sum y^2$ stands for the sum of the squared y-values and n for the number of data pairs.

So using a spreadsheet we get the following results:

x	y	x^2	y^2	xy
Volume (cm^3)	Frequency (Hz)			
1760	529	3097600	279841	931040
2040	566	4161600	320356	1154640
2440	473	5953600	223729	1154120
2550	461	6502500	212521	1175550
2730	465	7452900	216225	1269450
2740	532	7507600	283024	1457680
3010	484	9060100	234256	1456840
3080	527	9486400	277729	1623160
3370	488	11356900	238144	1644560
3740	485	13987600	235225	1813900
4910	478	24108100	228484	2346980
5090	434	25908100	188356	2209060
5090	468	25908100	219024	2382120
5380	449	28944400	201601	2415620
5850	425	34222500	180625	2486250
6730	389	45292900	151321	2617970
6990	421	48860100	177241	2942790
7960	416	63361600	173056	3311360
75460	**8490**	**375172600**	**4040758**	**34393090**

From the table we get the results:

$$\sum x = 75460 \quad \sum y = 8490 \quad \sum x^2 = 375172600 \quad \sum y^2 = 4040758 \quad \sum xy = 34393090$$

Hence

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 34393090 - \frac{75460 \times 8490}{18} = -1198877$$

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} = 375172600 - \frac{(75460)^2}{18} = 58827511$$

$$S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = 4040758 - \frac{(8490)^2}{18} = 36308$$

Then $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-1198877}{58827511 \text{ x } 36308} = -0.820\ (3sf)$

So this suggests that there is good correlation between the size of the sac and the frequency of the drumming noise. It is therefore quite plausible that females can use the pitch to identify a male with a large sac.

If we wanted to do this more formally, we could set up a **hypothesis** and conduct a significance test. We might also be interested in working out a **regression line** for this data.

References

McDonald, J.H. 2014. Handbook of Biological Statistics (3rd ed.). Sparky House Publishing, Baltimore, Maryland. [Online] December 2014. Available from **http://www.biostathandbook.com/spearman.html** [Accessed: 9/1/15]

Madsen, V., T.J.S. Balsby, T. Dabelsteen, and J.L. Osorno. 2004. Bimodal signaling of a sexually selected trait: gular pouch drumming in the magnificent frigatebird. Condor 106: 156-160.

Calcutt, M. 2006. Fregata magnificens [Online], Animal Diversity Web. Available from **http://animaldiversity.org/accounts/Fregata_magnificens/** [Accessed 12/1/15]