KING EDWARD VI SCHOOL

SHAKESPEARE'S SCHOOL

Year 12 Mathematics
A-Level Practice Booklet

About the Year 12 Mock Examinations

- There will be two 1.5-hour examinations which may contain content from any of the topics you have studied this year.
- You will be allowed a calculator for both papers. Note that AQA's guidance on calculator use is as follows and we shall also apply this in the Year 12 mocks:

> *"If students are asked to "show" or "justify" something, they may need to write more working, but otherwise our attitude will be to expect that students will use whatever calculator functionality they have available."*

- You should take these examinations seriously as they are an important indication of your progress during Year 12. However, they are not the only factor that your teacher will consider when predicting grades for UCAS and your performance across the year and at the beginning of Year 13 will also be considered.

About this Booklet

- These questions are taken from specimen papers with topics which we have not covered yet removed. Hence the question numbers will not be consecutive.
- Every effort has been made to ensure that only questions which have been covered on the Year 12 K.E.S. syllabus have been included. However, there may be some which have slipped through the net. If you are unsure, then please ask your teacher.
- To condense the size of the booklet, the answering space for the questions is not as it would be in an exam. Therefore, if your answer is too long then don't worry; you would have more room in the real thing.

Other Useful Resources

- Your teacher may have introduced you to drfrostmaths.com. If not, you can visit the site and sign up for yourself. It is good for practising techniques such as the chain rule, rules of logarithms or solving trigonometric equations. There are a large bank of questions and you input an answer and get immediately told whether it is right or wrong. Your teacher will also be able to see the questions you have attempted so if there are topics where you are regularly getting stuck you can discuss it with them.
- There are resources on Moodle, including notes interwoven with questions and topic tests from AQA.
- If you want more practice at exam-style questions you can look at past papers from the C1 – C4 modules for the old A-Level. You will know most, but not all, of the topics on those papers and you should bear this in mind when you encounter something unfamiliar.
- If you have a textbook, then you will also find explanations and additional exercises contained therein.

Topics Examined in the Year 12 Mock Examinations

Algebra

- Understand and use the laws of indices for all rational exponents
- Use and manipulate surds, including rationalising the denominator.
- Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown.
- Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.
- Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. Express solutions through correct use of 'and' and 'or', or through set notation.
- Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically
- Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem.
- Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only).

Functions

- Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of a linear function, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations.

- Understand and use proportional relationships and their graphs.
- Understand and use composite functions; inverse functions and their graphs.
- Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$, and combinations of these transformations.
- Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).
- Use of functions in modelling, including consideration of limitations and refinements of the models.

Trigonometry 1

- Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab \sin C$
- Work with radian measure, including use for arc length and area of sector.
- Know and use exact values of sin, cos and tan for multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$
- Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity.

- Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.
- Understand and use $tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$ and $sin^2\,\theta + cos^2\,\theta = 1$, $1 + tan^2\,\theta = sec^2\,\theta$ and $1 + cot^2\,\theta = csc^2\,\theta$
- Understand and use the standard small angle approximations of sine, cosine and tangent: $sin\,\theta \approx \theta$, $cos\,\theta \approx 1 - \frac{\theta^2}{2}$, $tan\,\theta = \theta$ where θ is in radians.

Trigonometry 2

- Understand and use double angle formulae; use of formulae for $sin(A \pm B)$, $cos(A \pm B)$ and $tan(A \pm B)$; understand geometrical proofs of these formulae.
- Understand and use expressions for $a\,cos\,\theta + b\,sin\,\theta$ a in the equivalent forms of $r\,cos(\theta \pm \alpha)$ or $r\,sin(\theta \pm \alpha)$
- Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle.
- Construct proofs involving trigonometric functions and identities.

Exponentials and Logarithms

- Know and use the function a^x and its graph, where a is positive.
- Know and use the function e^x and its graph.
- Know and use the definition of $log_a\,x$ as the inverse of a^x, where a is positive and $x \geq 0$
- Know and use the function $ln\,x$ and its graph.
- Know and use $ln\,x$ as the inverse function of e^x
- Understand and use the laws of logarithms:
    - $log_a\,x + log_a\,y = log_a\,xy$;
    - $log_a\,x - log_a\,y = log_a\,\frac{x}{y}$;
    - $klog_a x = log_a\,x^k$.
- Solve equations of the form $a^x = b$
- Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.
- Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.
- Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

Binomial Expansion

- Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations n! and nCr; link to binomial probabilities.
- Extend to any rational n, including its use for approximation; be aware that the expansion is valid for $\left|\frac{bx}{a}\right| < 1$ (proof not required).

Differentiation 1

- Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x and for $sin\, x$ and $cos\, x$
- Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.
- Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.
- Differentiate e^{kx} and a^{kx}, $sin\, kx$, $cos\, kx$, $tan\, kx$ and related sums, differences and constant multiples.
- Understand and use the derivative of $ln\, x$
- Apply differentiation to find maxima and minima and stationary points, points of inflection.
- Identify where functions are increasing or decreasing.
- Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.

Differentiation 2

- Apply differentiation to find gradients, tangents and normal.
- Differentiation from first principles of polynomials, $\sin x$ and $\cos x$

Integration

- Know and use the Fundamental Theorem of Calculus
- Integrate x^n and related sums, differences and constant multiples.
- Integrate e^{kx} , $\frac{1}{x}$, $sin\, kx$, $cos\, kx$ and related sums, differences and constant multiples.
- Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.
- Understand and use integration as the limit of a sum.
- Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively.
- Integrate using partial fractions that are linear in the denominator

Answer ALL questions. Write your answers in the spaces provided.

1. The line l passes through the points A (3, 1) and B (4, −2).

Find an equation for l.

(3)

(Total for Question 1 is 3 marks)

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2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6).

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

(Total for Question 2 is 4 marks)

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4.

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$

(4)

(Total for Question 4 is 6 marks)

5. Given that

$$f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0$$

show that $\displaystyle\int_1^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

(Total for Question 5 is 5 marks)

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6. Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

(Total for Question 6 is 4 marks)

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7. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

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8.

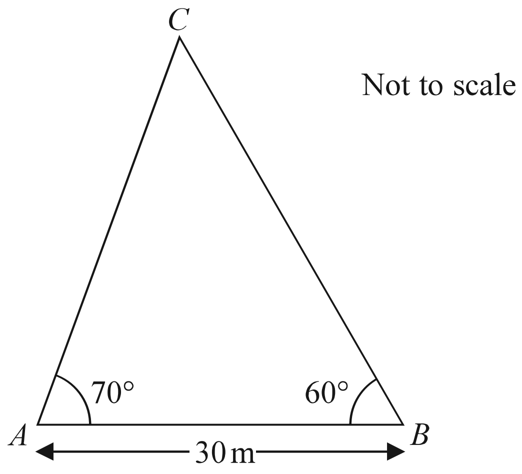

Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle $BAC = 70°$ and angle $ABC = 60°$,

(a) calculate the area of the lawn to 3 significant figures.

(4)

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

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9. Solve, for $360° \leqslant x < 540°$,

$$12\sin^2 x + 7\cos x - 13 = 0$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

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10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

(Total for Question 10 is 4 marks)

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12. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$

$$2^{2x} + 2^4 - 9(2^x) = 0$$

$$\text{Let} \quad 2^x = y$$

$$y^2 - 9y + 8 = 0$$

$$(y - 8)(y - 1) = 0$$

$$y = 8 \text{ or } y = 1$$

$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

(b) Find the exact solution to the equation.

(2)

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13. (a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

(b) Sketch the curve with equation

$$y = x^3 + 10x^2 + 25x$$

showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)

The point with coordinates $(-3, 0)$ lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a)$$

where a is a constant.

(c) Find the two possible values of a.

(3)

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14.

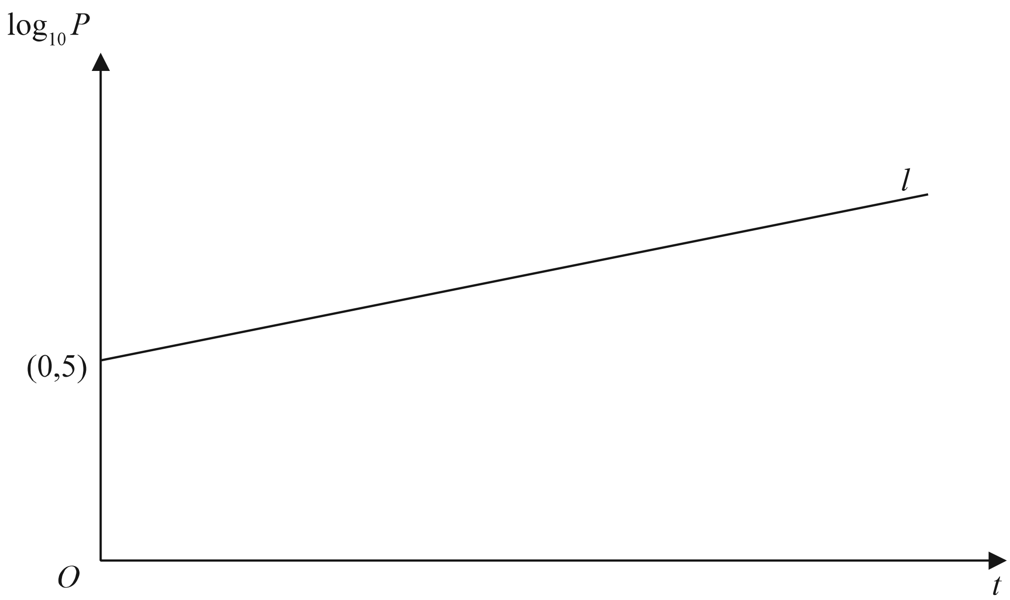

Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\dfrac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

Question 14 continued

(Total for Question 14 is 13 marks)

15.

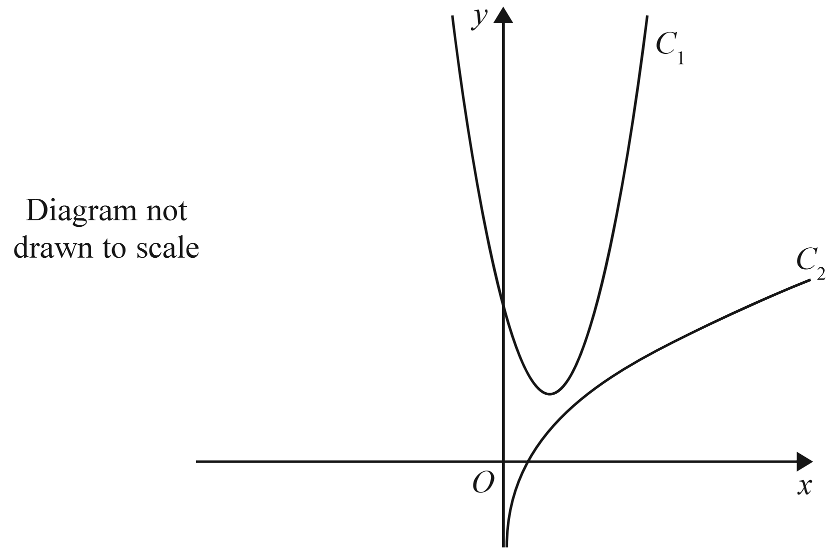

Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$.

The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(8)

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16.

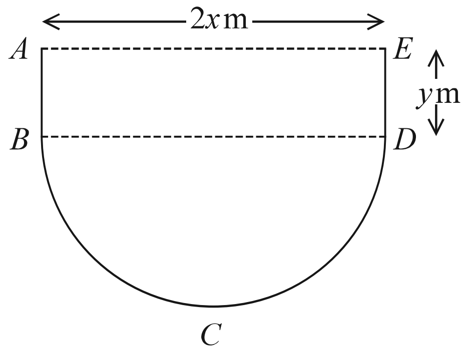


Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that $AE = 2x$ metres, $ED = y$ metres and the area of the pool is $250\,\text{m}^2$,

(a) show that the perimeter, *P* metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$

(2)

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures.

(4)

Paper 1: Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1 **Way 1**	Uses $y = mx + c$ with both (3, 1) and (4, −2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or **Way 2**	Uses $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$ with both (3, 1) and (4, −2)	M1	1.1b
	Gradient simplified to −3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or **Way 3**	Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find a, b or k in terms of one of them	M1	1.1b
	Obtains $a = 3b$, $k = -10b$ or $3k = -10a$	A1	1.1b
	Obtains $a = 3$, $b = 1$, $k = -10$ Or writes $3x + y - 10 = 0$ o.e.	A1	1.1b
		(3)	
			(7 marks)

Notes:

M1: Need correct use of the given coordinates

A1: Need fractions simplified to −3 (in ways 1 and 2)

A1: Need constants combined accurately

N.B. Answer left in the form $(y - 1) = -3(x - 3)$ or $(y - (-2)) = -3(x - 4)$ is awarded M1A1A0 as answers should be simplified by constants being collected

Note that a correct answer implies all three marks in this question

Question	Scheme	Marks	AOs
2	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
			(4 marks)

Notes:

M1: Differentiation implied by one correct term

A1: Correct differentiation

M1: Attempts to substitute $x = 5$ into their derived function

A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their f ′(5) so follow through slips in differentiation

Question	Scheme	Marks	AOs
4(a)	States or uses $f(+3)=0$	M1	1.1b
	$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so x $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + \ldots)$	M1	2.1
	$4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + 2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x So $x = 3$ is the only real root of $f(x) = 0$ *	A1*	2.4
		(4)	
		(6 marks)	

Notes:

(a)

M1: States or uses $f(+3) = 0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b)

M1: Needs to have $(x - 3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x - 3)(2x^2 + 1)$

M1: Considers their quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: A correct explanation

Question	Scheme	Marks	AOs
5	$f(x) = 2x + 3 + 12x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\frac{12}{x^2}\right)dx = x^2+3x-\frac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\frac{12(\sqrt{2})}{2\times 2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes:

B1: Correct function with numerical powers
M1: Allow for raising power by one. $x^n \rightarrow x^{n+1}$
A1: Correct three terms
M1: Substitutes limits and rationalises denominator
A1*: Completely correct, no errors seen

Question	Scheme	Marks	AOs
6	Considers $\dfrac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $= \dfrac{6xh+3h^2}{h} = 6x+3h$ or $\dfrac{6x\delta x+3(\delta x)^2}{\delta x} = 6x+3\delta x$	A1	1.1b
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ *	A1*	2.5
			(4 marks)

Notes:

B1: Gives correct fraction as in the scheme above or $\dfrac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2 = 3x^2 + 6x\delta x + 3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Uses Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

Question	Scheme	Marks	AOs
7(a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right)+\binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128 + ...$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... -224x + ...$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... + ... + 168x^2 (+ ...)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Need correct binomial coefficient with correct power of 2 and correct power of x. Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent

B1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

(b)

B1: Needs a full explanation i.e. to state $x = 0.01$ and that this would be substituted and that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

Question	Scheme		Marks	AOs
8(a)	**Way 1** Finds third angle of triangle and uses or states $\frac{x}{\sin 60°} = \frac{30}{\sin "50°"}$	**Way 2** Finds third angle of triangle and uses or states $\frac{y}{\sin 70°} = \frac{30}{\sin "50°"}$	M1	2.1
	So $x = \frac{30 \sin 60°}{\sin 50°}$ $(= 33.9)$	So $y = \frac{30 \sin 70°}{\sin 50°}$ $(= 36.8)$	A1	1.1b
	Area = $\frac{1}{2} \times 30 \times x \times \sin 70\ °$ or $\frac{1}{2} \times 30 \times y \times \sin 60$		M1	3.1a
	= 478 m^2		A1ft	1.1b
			(4)	
(b)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1	3.2b
			(1)	
				(5 marks)

Notes:

(a)

M1: Uses sine rule with their third angle to find one of the unknown side lengths

A1: Finds expression for, or value of either side length

M1: Completes method to find area of triangle

A1ft: Obtains a correct answer for their value of x or their value of y

(b)

B1: As information given in the question may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate

Question	Scheme	Marks	AOs
9	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x = 430.5°, 435.5°$	A1	1.1b
			(5 marks)

Notes:

M1: Uses correct identity

A1: Correct three term quadratic

M1: Solves their three term quadratic to give values for $\cos x$. (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

A1: Two correct answers in the given domain

Question	Scheme	Marks	AOs
10	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k - 3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes:

B1: Explains why $k = 0$ gives no real roots
M1: Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark
M1: Attempts solution of quadratic inequality
A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Question	Scheme		Marks	AOs
12(a)	$2^{2x} + 2^4$ is wrong in line 2 - it should be $2^{2x} \times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	**Way 1:** $2^{2x+4} - 9(2^x) = 0$ $2^{2x} \times 2^4 - 9(2^x) = 0$ Let $2^x = y$ $16y^2 - 9y = 0$	**Way 2:** $(2x+4)\log 2 - \log 9 - x\log 2 = 0$	M1	2.1
	$y = \frac{9}{16}$ or $y = 0$ So $x = \log_2\left(\frac{9}{16}\right)$ or $\frac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer	$x = \frac{\log 9}{\log 2} - 4$ o.e.	A1	1.1b
			(2)	
				(4 marks)

Notes:

(a)

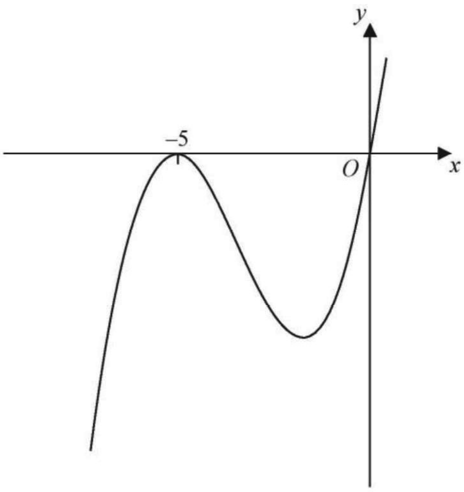
B1: Lists error in line 2 (as above)

B1: Lists error in line 4 (as above)

(b)

M1: Correct work with powers reaching this equation

A1: Correct answer here – there are many exact equivalents

Question	Scheme	Marks	AOs
13(a)	$x^3+10x^2+25x = x(x^2+10x+25)$	M1	1.1b
	$= x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
			(7 marks)

Notes:

(a)
M1: Takes out factor x
A1: Correct factorisation – allow $x(x + 5)(x + 5)$

(b)
M1: Correct shape
A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c)
M1: May be implied by one of the correct answers for a or by a statement
A1ft: ft from their cubic as long as it meets the x-axis only twice
A1ft: ft from their cubic as long as it meets the x-axis only twice

Question	Scheme		Marks	AOs
14(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	**Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So a = 100 000 **or** b = 1.0116		A1	1.1b
	Both a = 100 000 **and** b = 1.0116 (awrt 1.01)		A1	1.1b
			(4)	
(c)(i)	The initial population		B1	3.4
(c)(ii)	The proportional increase of population each year		B1	3.4
			(2)	
(d)(i)	300000 to nearest hundred thousand		B1	3.4
(d)(ii)	Uses $200\,000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	

Question 14 continued
Notes:
(a) **M1:** Uses a linear equation to relate log P and t **A1:** Correct use of gradient and intercept to give a correct line equation
(b) **M1:** **<u>Way 1</u>:** Uses logs correctly to give log equation; **<u>Way 2:</u>** Uses powers correctly to "undo" log equation and expresses as product of two powers **M1:** **<u>Way 1</u>:** Identifies log b or log a or both; **<u>Way 2:</u>** Identifies a or b as powers of 10 **A1:** Correct value for a or b **A1:** Correct values for both
(c)(i) **B1:** Accept equivalent answers e.g. The population at $t = 0$ **(c)(ii)** **B1:** So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year
(d)(i) **B1:** cao **(d)(ii)** **M1:** As in the scheme **A1ft:** On their values of a and b with correct log work
(e) **B2:** As given in the scheme – any two valid reasons

Question	Scheme	Marks	AOs
15	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is –2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

Notes:

M1: Differentiates correctly

M1: Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

M1: Uses negative reciprocal gradient

A1: Correct equation for normal

M1: Attempts to eliminate y to find an equation in x

M1: Attempts to solve their equation using exp

M1: Uses their x value to find y

A1: Any correct exact form

Question	Scheme	Marks	AOs
16(a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1	2.1
	Obtain $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P=2x+2y+\pi x$ with their y substituted	M1	2.1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{\mathrm{d}P}{\mathrm{d}x}$; $x^{-1}\to x^{-2}$	M1	3.4
	$\frac{\mathrm{d}P}{\mathrm{d}x}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1	1.1b
	Sets $\frac{\mathrm{d}P}{\mathrm{d}x}=0$ and proceeds to $x=$	M1	1.1b
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8 M	A1	1.1b
		(4)	
			(10 marks)

Question 16 continued
Notes:
(a) **B1:** Correct area equation **M1:** Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P **M1:** Use correct equation for perimeter with their y substituted **A1*:** Completely correct solution to obtain and state printed answer
(b) **M1:** States $x > 0$ and $y > 0$ and uses their expression from (a) to form inequality **A1*:** Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly
(c) **M1:** Attempt to differentiate P (deals with negative power of x correctly) **A1:** Correct differentiation **M1:** Sets derived function equal to zero and obtains $x =$ **A1:** The value of x may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$) Need to see awrt 59.8 M with units included for the perimeter

Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

2.

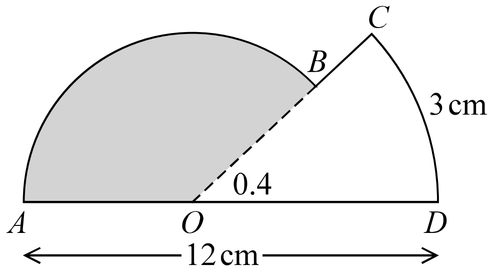


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, ∠*COD* = 0.4 radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB.*

(3)

(Total for Question 2 is 5 marks)

4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

(Total for Question 4 is 4 marks)

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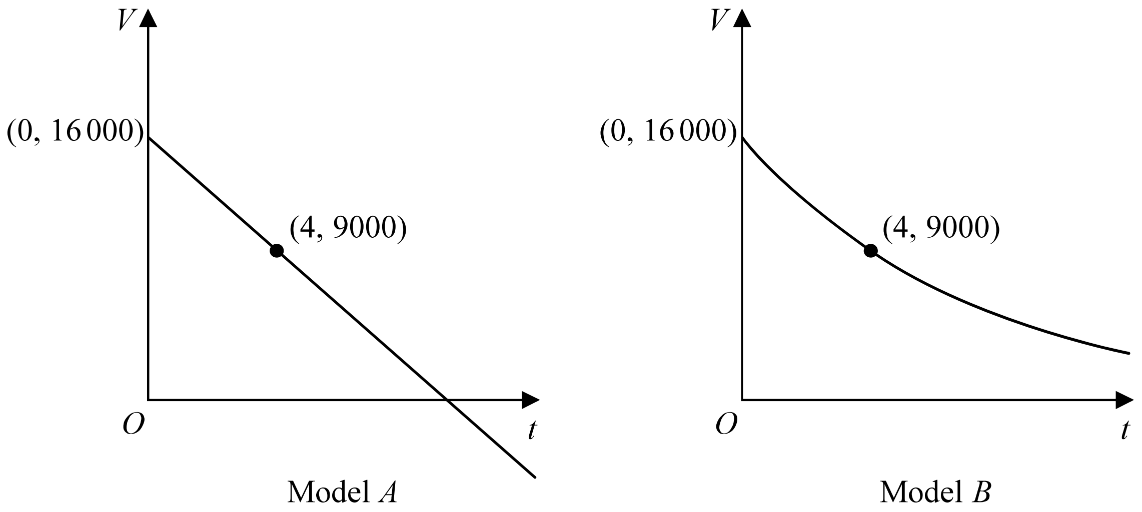
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

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Question 6 continued

(Total for Question 6 is 7 marks)

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total for Question 9 is 5 marks)

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10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total for Question 10 is 5 marks)

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.
(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.
(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.
(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.
(2)

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12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

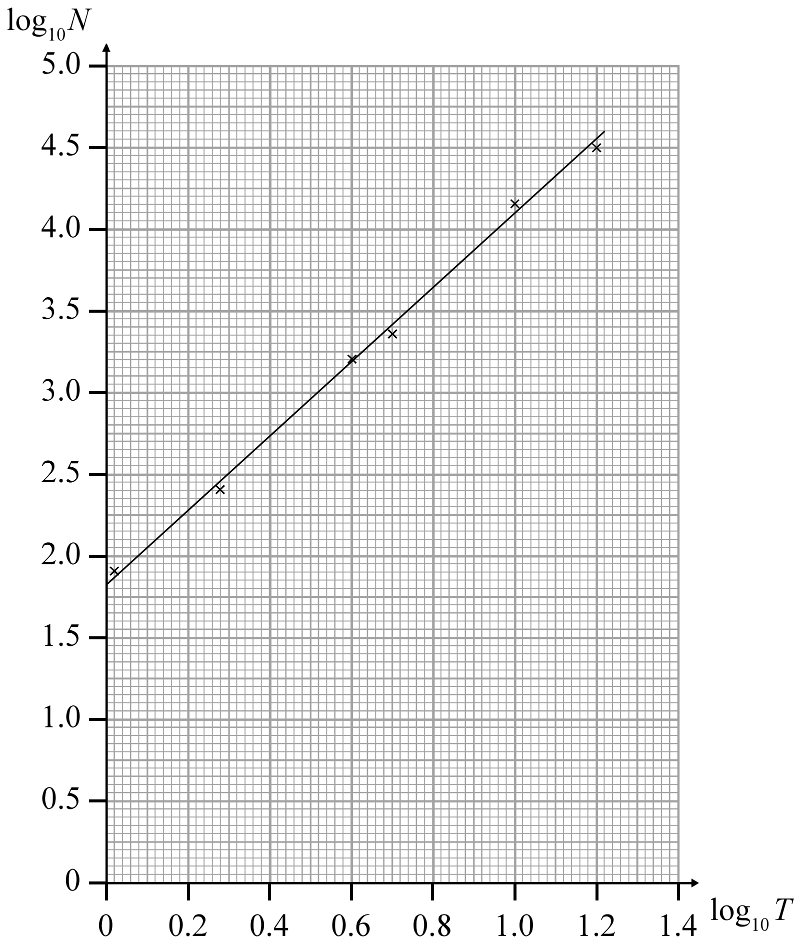

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

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Question 12 continued

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15.

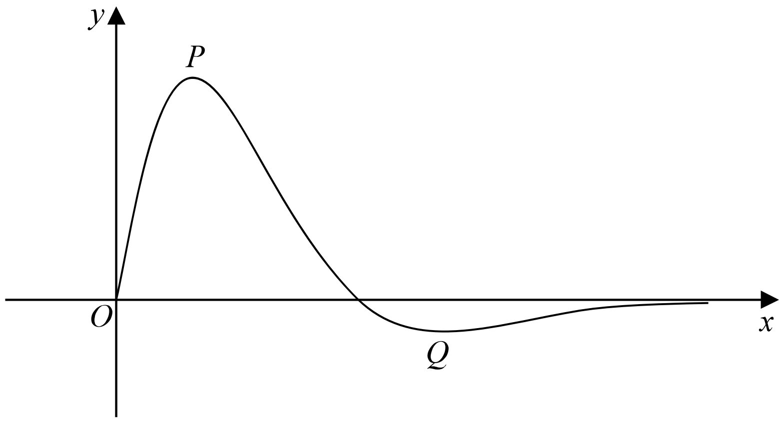

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$.

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

Paper 1: Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12\times 2^3 - 24\times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36\times 2^2 - 48\times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
			(7 marks)

Notes:

(a)(i)

M1: Differentiates to a cubic form

A1: $\frac{dy}{dx} = 12x^3 - 24x^2$

(a)(ii)

A1ft: Achieves a correct $\frac{d^2y}{dx^2}$ for their $\frac{dy}{dx} = 36x^2 - 48x$

(b)

M1: Substitutes $x = 2$ into their $\frac{dy}{dx}$

A1: Shows $\frac{dy}{dx}$ = 0 and states "hence there is a stationary point" All aspects of the proof must be correct

(c)

M1: Substitutes $x = 2$ into their $\frac{d^2y}{dx^2}$

Alternatively calculates the gradient of C either side of $x = 2$

A1ft: For a correct calculation, a valid reason and a correct conclusion.

Follow through on an incorrect $\frac{d^2y}{dx^2}$

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5$ cm	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12 - 7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: Attempts to use the correct formula $s = r\theta$ with $s = 3$ and $\theta = 0.4$

A1: OD = 7.5 cm (An answer of 7.5cm implies the use of a correct formula and scores both marks)

(b)

M1: $AOB = \pi - 0.4$ may be implied by the use of AOB = awrt 2.74 or uses radius is (12 – their '7.5')

M1: Follow through on their radius (12 – their OD) and their angle

A1ft: Allow awrt 27.8 cm^2. (Answer 27.75862562). Follow through on their (12 – their '7.5')
Note: Do not follow through on a radius that is negative.

Question	Scheme	Marks	AOs
4	Writes $\int \frac{t+1}{t}\,\mathrm{d}t = \int 1+\frac{1}{t}\,\mathrm{d}t$ and attempts to integrate	M1	2.1
	$= t + \ln t \ (+c)$	M1	1.1b
	$(2a + \ln 2a) - (a + \ln a) = \ln 7$	M1	1.1b
	$a = \ln\frac{7}{2}$ with $k = \frac{7}{2}$	A1	1.1b
		(4 marks)	

Notes:

M1: Attempts to divide each term by t or alternatively multiply each term by t^{-1}

M1: Integrates each term and knows $\int \frac{1}{t}\,\mathrm{d}t = \ln t$. The + c is not required for this mark

M1: Substitutes in both limits, subtracts and sets equal to ln7

A1: Proceeds to $a = \ln\frac{7}{2}$ and states $k = \frac{7}{2}$ or exact equivalent such as 3.5

Question	Scheme	Marks	AOs
6 (a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t=10, V=-1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t=3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
			(7 marks)

Notes:

(a)(i)
B1: 10750 barrels
(a)(ii)
B1: See scheme

(b)(i)
M1: Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.
dM1: Uses both $(0,16000)$ and $(4,9000)$ in their model.
With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$
With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$
With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.
M1: Uses a correct method to find all constants in the model.
A1: Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example
$V = 16000e^{-0.144t}$ $\quad V = 16000 \times (0.866)^t$ $\quad V = 15800e^{-0.146t} + 200$

(b)(ii)
B1ft: Follow through on their exponential model

Question	Scheme	Marks	AOs
9(a)	$\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv\dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\text{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta+\cot\theta=1\Rightarrow\sin 2\theta=2$ AND no real solutions as $-1\leqslant\sin 2\theta\leqslant 1$	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Writes $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

A1: Achieves a correct intermediate answer of $\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

M1: Uses the double angle formula $\sin 2\theta=2\sin\theta\cos\theta$

A1*: Completes proof with no errors. This is a given answer.

Note: There are many alternative methods. For example

$\tan\theta+\cot\theta\equiv\tan\theta+\dfrac{1}{\tan\theta}\equiv\dfrac{\tan^2\theta+1}{\tan\theta}\equiv\dfrac{\sec^2\theta}{\tan\theta}\equiv\dfrac{1}{\cos^2\theta\times\dfrac{\sin\theta}{\cos\theta}}\equiv\dfrac{1}{\cos\theta\times\sin\theta}$ then as the main scheme.

(b)

B1: Scored for sight of $\sin 2\theta=2$ and a reason as to why this equation has no real solutions.
Possible reasons could be $-1\leqslant\sin 2\theta\leqslant 1$........and therefore $\sin 2\theta\neq 2$
or $\sin 2\theta=2\Rightarrow 2\theta=\arcsin 2$ which has no answers as $-1\leqslant\sin 2\theta\leqslant 1$

Question	Scheme	Marks	AOs
10	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0,\ \dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

Notes:

B1: States or implies that the gradient of the chord is $\dfrac{\sin(\theta+h)-\sin\theta}{h}$ or similar such as $\dfrac{\sin(\theta+\delta\theta)-\sin\theta}{\theta+\delta\theta-\theta}$ for a small h or $\delta\theta$

M1: Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ or $\delta\theta$

A1: Obtains $\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$ or equivalent

M1: Writes their expression in terms of $\dfrac{\sin h}{h}$ and $\dfrac{\cos h-1}{h}$

A1*: Uses correct language to explain that $\dfrac{dy}{d\theta}=\cos\theta$

For this method they should use all of the given statements $h\to 0,\ \dfrac{\sin h}{h}\to 1$, $\dfrac{\cos h-1}{h}\to 0$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$

and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$

Question	Scheme	Marks	AOs
10alt	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Sets $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\dfrac{\sin\left(\theta+\frac{h}{2}+\frac{h}{2}\right)-\sin\left(\theta+\frac{h}{2}-\frac{h}{2}\right)}{h}$ and uses the compound angle identity for sin(*A* + *B*) and sin(*A* – *B*) with $A=\theta+\frac{h}{2},\quad B=\frac{h}{2}$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=$ $\dfrac{\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)+\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]-\left[\sin\left(\theta+\frac{h}{2}\right)\cos\left(\frac{h}{2}\right)-\cos\left(\theta+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)\right]}{h}$	A1	1.1b
	$=\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\cos\left(\theta+\frac{h}{2}\right)$	M1	2.1
	Uses $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ and $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ Therefore the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$ *	A1*	2.5
		(5 marks)	

Additional notes:

A1*: Uses correct language to explain that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$. For this method they should use the (adapted) given statement $h\to 0,\ \frac{h}{2}\to 0$ hence $\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\to 1$ with $\cos\left(\theta+\frac{h}{2}\right)\to\cos\theta$ meaning that the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and therefore the gradient of the chord $\to$ gradient of the curve $\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta$

Question	Scheme	Marks	AOs
11(a)	Sets $H = 0 \Rightarrow 1.8 + 0.4d - 0.002d^2 = 0$	M1	3.4
	Solves using an appropriate method, for example $d = \dfrac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(1.8)}}{2 \times -0.002}$	dM1	1.1b
	Distance = awrt 204 (m) only	A1	2.2a
		(3)	
(b)	States the initial height of the arrow above the ground.	B1	3.4
		(1)	
(c)	$1.8 + 0.4d - 0.002d^2 = -0.002(d^2 - 200d) + 1.8$	M1	1.1b
	$= -0.002((d-100)^2 - 10000) + 1.8$	M1	1.1b
	$= 21.8 - 0.002(d-100)^2$	A1	1.1b
		(3)	
(d)	(i) 22.1 metres	B1ft	3.4
	(ii) 100 metres	B1ft	3.4
		(2)	
		(9 marks)	

Notes:

(a)

M1: Sets $H = 0 \Rightarrow 1.8 + 0.4d - 0.002d^2 = 0$

M1: Solves using formula, which if stated must be correct, by completing square (look for $(d-100)^2 = 10900 \Rightarrow d = ..$) or even allow answers coming from a graphical calculator

A1: Awrt 204 m only

(b)

B1: States it is the initial height of the arrow above the ground. Do not allow " it is the height of the archer"

(c)

M1: Score for taking out a common factor of -0.002 from at least the d^2 and d terms

M1: For completing the square for their $(d^2 - 200d)$ term

A1: $= 21.8 - 0.002(d-100)^2$ or exact equivalent

(d)

B1ft: For their '21.8+0.3' =22.1m

B1ft: For their 100m

Question	Scheme	Marks	AOs
12 (a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
		(9 marks)	

Question 12 continued
Notes:
(a) **M1:** Takes logs of both sides and shows the addition law **M1:** Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$
(b) **M1:** Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$ **M1:** Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$ **M1:** Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work. There is an alternative to this using a graphical approach. **M1**: Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$ **M1**: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48" Accept $\log_{10} N \approx 2.9$ **M1:** Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$ **A1:** Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work
(c) **M1** For using N = 1000000 and stating that $\log_{10} N = 6$ **A1:** Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula. **M1:** Use N = 1000000 in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\dfrac{1000000}{63}\right)}{2.3} \approx 1.83$. **A1:** The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d) **B1:** Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at T=1

Question	Scheme	Marks	AOs
15(a)	Attempts to differentiate using the quotient rule or otherwise	M1	2.1
	$\mathrm{f}'(x)=\dfrac{\mathrm{e}^{\sqrt{2}x-1}\times 8\cos 2x-4\sin 2x\times\sqrt{2}\mathrm{e}^{\sqrt{2}x-1}}{\left(\mathrm{e}^{\sqrt{2}x-1}\right)^2}$	A1	1.1b
	Sets $\mathrm{f}'(x)=0$ and divides/ factorises out the $\mathrm{e}^{\sqrt{2}x-1}$ terms	M1	2.1
	Proceeds via $\dfrac{\sin 2x}{\cos 2x}=\dfrac{8}{4\sqrt{2}}$ to $\Rightarrow\tan 2x=\sqrt{2}$ *	A1*	1.1b
		(4)	
(b)	(i) Solves $\tan 4x=\sqrt{2}$ and attempts to find the 2nd solution	M1	3.1a
	$x=1.02$	A1	1.1b
	(ii) Solves $\tan 2x=\sqrt{2}$ and attempts to find the 1st solution	M1	3.1a
	$x=0.478$	A1	1.1b
		(4)	
		(8 marks)	

Notes:

(a)

M1: Attempts to differentiate by using the quotient rule with $u=4\sin 2x$ and $v=\mathrm{e}^{\sqrt{2}x-1}$ or alternatively uses the product rule with $u=4\sin 2x$ and $v=\mathrm{e}^{1-\sqrt{2}x}$

A1: For achieving a correct $\mathrm{f}'(x)$. For the product rule

$\mathrm{f}'(x)=\mathrm{e}^{1-\sqrt{2}x}\times 8\cos 2x+4\sin 2x\times-\sqrt{2}\mathrm{e}^{1-\sqrt{2}x}$

M1: This is scored for cancelling/ factorising out the exponential term. Look for an equation in just $\cos 2x$ and $\sin 2x$

A1*: Proceeds to $\tan 2x=\sqrt{2}$. This is a given answer.

(b) (i)

M1: Solves $\tan 4x=\sqrt{2}$ attempts to find the 2nd solution. Look for $x=\dfrac{\pi+\arctan\sqrt{2}}{4}$

Alternatively finds the 2nd solution of $\tan 2x=\sqrt{2}$ and attempts to divide by 2

A1: Allow awrt $x=1.02$. The correct answer, with no incorrect working scores both marks

(b)(ii)

M1: Solves $\tan 2x=\sqrt{2}$ attempts to find the 1st solution. Look for $x=\dfrac{\arctan\sqrt{2}}{2}$

A1: Allow awrt $x=0.478$. The correct answer, with no incorrect working scores both marks

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

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2. Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation

$$\cos\theta = 2\sin\theta$$

The attempts of two of the students are shown below.

Student *A*	Student *B*
$\cos\theta = 2\sin\theta$ $\tan\theta = 2$ $\theta = 63.4°$	$\cos\theta = 2\sin\theta$ $\cos^2\theta = 4\sin^2\theta$ $1 - \sin^2\theta = 4\sin^2\theta$ $\sin^2\theta = \frac{1}{5}$ $\sin\theta = \pm\frac{1}{\sqrt{5}}$ $\theta = \pm 26.6°$

(a) Identify an error made by student *A*.

(1)

Student *B* gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(Total for Question 2 is 3 marks)

3. Given $y = x(2x+1)^4$, show that

$$\frac{dy}{dx} = (2x+1)^n(Ax+B)$$

where n, A and B are constants to be found.

(4)

(Total for Question 3 is 4 marks)

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

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5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for Question 5 is 4 marks)

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6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0, \quad (a \neq 0)$ has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive. **(2)**				
(ii) If $ax > b$ then $x > \dfrac{b}{a}$ **(2)**				
(iii) The difference between consecutive square numbers is odd. **(2)**				

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4 - x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

9. Given that A is constant and

$$\int_1^4 \left(3\sqrt{x} + A\right)dx = 2A^2$$

show that there are exactly two possible values for A.

(5)

(Total for Question 9 is 5 marks)

11.

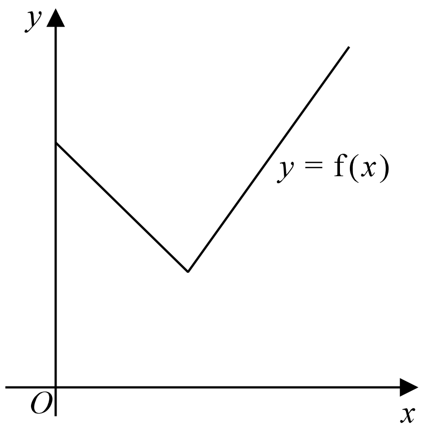

Figure 2

Figure 2 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)

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13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

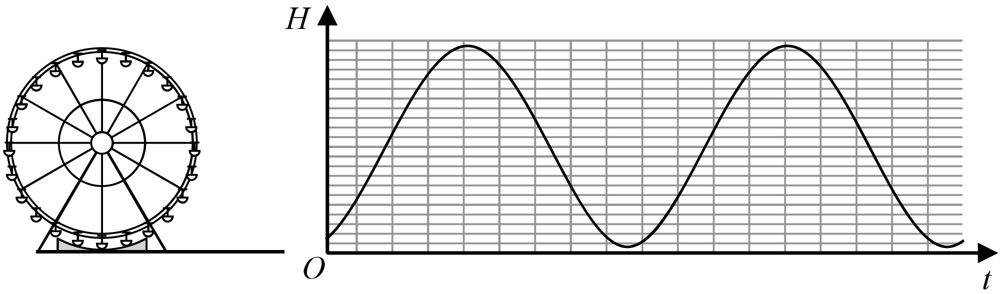

Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

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Question 13 continued

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

15.

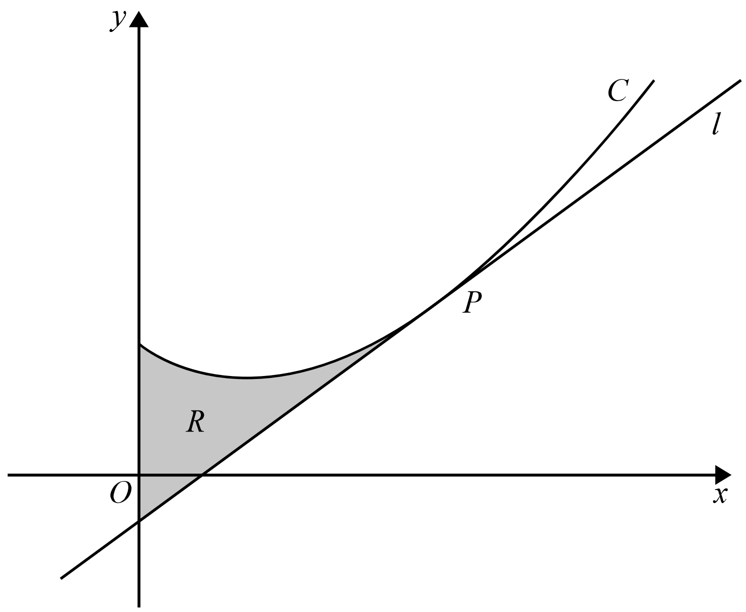

Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, \ x \geqslant 0$$

The point P with coordinates (4, 15) lies on C.

The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

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Question 15 continued

(Total for Question 15 is 10 marks)

Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $f(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
		(3 marks)	

Notes:

M1: Selects a suitable method given that $(x+2)$ is a factor of $f(x)$
Accept either setting $f(-2)=0$ or attempted division of $f(x)$ by $(x+2)$

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=..\Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\frac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
		(3 marks)	

Notes:

(a)

B1: Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\frac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ±5

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n=3, A=10, B=1$

Question	Scheme	Marks	AOs
4 (a)	$\mathrm{gf}(x) = 3\ln \mathrm{e}^x$	M1	1.1b
	$= 3x, \ (x \in \mathbb{R})$	A1	1.1b
		(2)	
(b)	$\mathrm{gf}(x) = \mathrm{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln \mathrm{e}^x$
An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $\mathrm{e}^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$
or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \leqslant 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05\times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$	M1	2.1
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -0.05 \times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
		(4 marks)	

Notes:

(a)

M1: Substitutes $t = 0.5$ into $m = 25\mathrm{e}^{-0.05t} \Rightarrow m = 25\mathrm{e}^{-0.05\times 0.5}$

A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks

(b)

M1: Applies the rule $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kx}\right) = k\,\mathrm{e}^{kx}$ in this context by stating or using $\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right) = \pm C\,\mathrm{e}^{-0.05t}$

A1: $\frac{\mathrm{d}m}{\mathrm{d}t} = -0.05 \times 25\mathrm{e}^{-0.05t} = -0.05m \Rightarrow k = -0.05$

Question	Scheme	Marks	AOs
6(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2\geqslant 0 \Rightarrow (x-3)^2+1\geqslant 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true. This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$; if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^2+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^2+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used	B1	2.4
		(1)	
			(5 marks)

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^2+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^2+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^2+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used

Question	Scheme		Marks	AOs
9	$\int\left(3x^{0.5}+A\right)\mathrm{d}x=2x^{1.5}+Ax(+c)$		M1 A1	3.1a 1.1b
	Uses limits and sets = $2A^2 \Rightarrow (2\times8+4A)-(2\times1+A)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
				(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax(+c)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
11 (a)	$f(x) \geqslant 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \leqslant 11$	M1	2.2a
	$\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$	A1	2.5
		(2)	
		(6 marks)	

Notes:

(a)

B1: $f(x) \geqslant 5$ Also allow $f(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \leqslant 11$

A1: Correct solution only $\{k : k \in \mathbb{R}, 5<k \leqslant 11\}$

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° =$ their '$-19.47°$'

A1ft: $\theta = 5.26°$ but allow a follow through on their '$-19.47°$'

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\frac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\frac{540-"16.70"}{80}=(6.54)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha=\pm\frac{3}{10}$

A1: $\alpha=16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11+$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t+"16.70"=540$. Follow through on their 16.70

M1: Solves their $80t+"16.70"=540$ correctly to find t

A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant	M1	2.1
	Radius $= 4.30$ cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2}$ $\Rightarrow$ Height $= 8.60$ cm	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
			9 marks

Notes:

(a)

B1: Uses the correct volume formula with V =500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi r h$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k, k$ is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question	Scheme	Marks	AOs
15	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{15}{2}x^{\frac{1}{2}}-9$	M1 A1	3.1a 1.1b
	Substitutes $x=4\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y-15=6(x-4)$	M1	2.1
	Equation of l is $y=6x-9$	A1	1.1b
	Area $R=\int_0^4\left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,\mathrm{d}x$	M1	3.1a
	$=\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x\right]_0^4=2\times4^{\frac{5}{2}}-\frac{15}{2}\times4^2+20\times4-0$	M1	2.1
	Area of $R=24$ *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
(10 marks)			

Question 15 continued
Notes:

M1: Differentiates $5x^{\frac{3}{2}}-9x+11$ to a form $Ax^{\frac{1}{2}}+B$

A1: $\frac{dy}{dx}=\frac{15}{2}x^{\frac{1}{2}}-9$ but may not be simplified

M1: Substitutes $x=4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)-(6x-9)\,dx$ following through on their $y=6x-9$

Look for a form $Ax^{\frac{5}{2}}+Bx^2+Cx$

A1: $=\left[2x^{\frac{5}{2}}-\frac{15}{2}x^2+20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve = $\int_0^4 \left(5x^{\frac{3}{2}}-9x+11\right)=\left[Ax^{\frac{5}{2}}+Bx^2+Cx\right]_0^4$

A1: $=\left[2x^{\frac{5}{2}}-\frac{9}{2}x^2+11x\right]_0^4=36$

M1: This requires a full method with all triangles found using a correct method

Look for Area R = their $36-\frac{1}{2}\times15\times\left(4-\text{their }\frac{3}{2}\right)+\frac{1}{2}\times\text{their }9\times\text{their }\frac{3}{2}$

Section A

Answer **all** questions in the spaces provided.

1 The curve $y = \sqrt{x}$ is translated onto the curve $y = \sqrt{x + 4}$

The translation is described by a vector.

Find this vector.

Circle your answer.

[1 mark]

$\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -4 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 4 \end{bmatrix}$ $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$

3 (a) Write down the value of p and the value of q given that:

3 (a) (i) $\sqrt{3} = 3^p$

[1 mark]

3 (a) (ii) $\frac{1}{9} = 3^q$

[1 mark]

3 (b) Find the value of x for which $\sqrt{3} \times 3^x = \frac{1}{9}$

[2 marks]

4 Show that $\frac{5\sqrt{2}+2}{3\sqrt{2}+4}$ can be expressed in the form $m+n\sqrt{2}$, where m and n are integers.

[3 marks]

5 Jessica, a maths student, is asked by her teacher to solve the equation $\tan x = \sin x$, giving all solutions in the range $0° \leq x \leq 360°$

The steps of Jessica's working are shown below.

$$\tan x = \sin x$$

		$\tan x = \sin x$	
Step 1	$\Rightarrow$	$\dfrac{\sin x}{\cos x} = \sin x$	Write $\tan x$ as $\dfrac{\sin x}{\cos x}$
Step 2	$\Rightarrow$	$\sin x = \sin x \cos x$	Multiply by $\cos x$
Step 3	$\Rightarrow$	$1 = \cos x$	Cancel $\sin x$
	$\Rightarrow$	$x = 0°$ or $360°$	

The teacher tells Jessica that she has not found all the solutions because of a mistake.

Explain why Jessica's method is not correct.

[2 marks]

6 A parallelogram has sides of length 6 cm and 4.5 cm.

The larger interior angles of the parallelogram have size α

Given that the area of the parallelogram is 24 cm^2, find the exact value of $\tan\alpha$

[4 marks]

7 Determine whether the line with equation $2x + 3y + 4 = 0$ is parallel to the line through the points with coordinates (9, 4) and (3, 8).

[4 marks]

Turn over for the next question

8 (a) Find the first **three** terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$

[3 marks]

8 (b) Carly has lost her calculator. She uses the first three terms, in ascending powers of x, of the expansion of $(1-2x)^{10}$ to evaluate 0.998^{10}

Find Carly's value for 0.998^{10} and show that it is correct to **five** decimal places.

[3 marks]

9 (a) Given that $f(x) = x^2 - 4x + 2$, find $f(3 + h)$

Express your answer in the form $h^2 + bh + c$, where b and $c \in \mathbb{Z}$.

[2 marks]

9 (b) The curve with equation $y = x^2 - 4x + 2$ passes through the point *P*(3, –1) and the point *Q* where $x = 3 + h$.

Using differentiation from first principles, find the gradient of the tangent to the curve at the point *P*.

[3 marks]

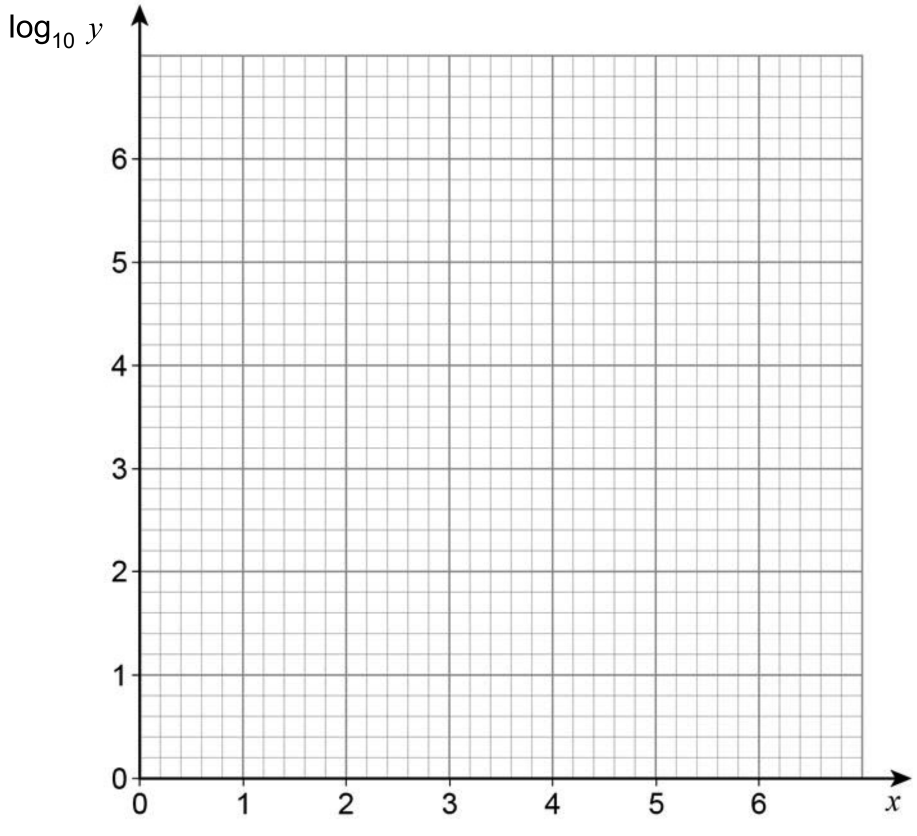
10 A student conducts an experiment and records the following data for two variables, x and y.

x	1	2	3	4	5	6
y	14	45	130	1100	1300	3400
$\log_{10} y$						

The student is told that the relationship between x and y can be modelled by an equation of the form $y = kb^x$

10 (a) Plot values of $\log_{10} y$ against x on the grid below.

[2 marks]



10 (b) State, with a reason, which value of y is likely to have been recorded incorrectly.

[1 mark]

___

___

___

10 (c) By drawing an appropriate straight line, find the values of k and b.

[4 marks]

Turn over for the next question

11 Chris claims that, “for any given value of x, the gradient of the curve $y = 2x^3 + 6x^2 - 12x + 3$ is always greater than the gradient of the curve $y = 1 + 60x - 6x^2$ ”.

Show that Chris is wrong by finding all the values of x for which his claim is **not** true.

[7 marks]

12 A curve has equation $y = 6x\sqrt{x} + \frac{32}{x}$ for $x > 0$

12 (a) Find $\frac{dy}{dx}$

[4 marks]

12 (b) The point *A* lies on the curve and has x-coordinate 4

Find the coordinates of the point where the tangent to the curve at *A* crosses the x-axis.

[5 marks]

END OF SECTION A
TURN OVER FOR SECTION B

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	$\begin{bmatrix} -4 \\ 0 \end{bmatrix}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)(i)	States correct value of p	AO1.2	B1	$p = \frac{1}{2}$
(a)(ii)	States correct value of q	AO1.2	B1	$q = -2$
(b)	Uses valid method to find x, PI	AO1.1a	M1	$\frac{1}{2} + x = -2$
	Obtains correct x, ACF	AO1.1b	A1	$x = -2.5$
	Total		**4**	
4	Multiplies numerator and denominator by the conjugate surd of the denominator	AO1.1a	M1	$\frac{(5\sqrt{2}+2)(3\sqrt{2}-4)}{(3\sqrt{2}+4)(3\sqrt{2}-4)}$
	Obtains **either** numerator **or** denominator correctly, in expanded or simplified form	AO1.1b	A1	$= \frac{30-20\sqrt{2}+6\sqrt{2}-8}{2}$ $= \frac{22-14\sqrt{2}}{2}$
	Constructs rigorous mathematical argument to show the required result Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips NMS = 0	AO2.1	R1	$= 11-7\sqrt{2}$
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Demonstrates a clear understanding that $\sin x = 0$ is a solution, and that this has not been properly taken into account.	AO2.3	R1	$\sin x = 0$ leads to a solution, but when she cancelled $\sin x$ she effectively assumed it was not equal to 0 and hence lost a number of solutions.
	Explains that cancelling $\sin x$ is not allowed if it is zero / only allowed if it is non-zero	AO2.4	E1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Translates given information into an equation by using the formula for the area of triangle or parallelogram to form a correct equation	AO3.1a	M1	AB × AD × $\sin\alpha = 24$ hence 6 × 4.5 × $\sin\alpha = 24$
	Rearranges 'their' equation to obtain a correct value of $\sin\alpha$	AO1.1b	A1F	$\sin\alpha = \frac{24}{27} = \frac{8}{9}$
	Uses 'their' $\sin\alpha$ value to identify an appropriate right-angled triangle **or** uses identities **and** deduces exact ratio of $\tan\alpha$ – positive or negative Condone only positive ratio seen	AO2.2a	M1	Sides of right angled triangle are 8, 9 and $\sqrt{17}$ Hence $\tan\alpha = \pm\frac{8}{\sqrt{17}}$
	Relates back to mathematical context of problem and hence chooses negative ratio – accept any equivalent exact form FT 'their' tan values for obtuse α	AO3.2a	A1F	α is one of the largest angles and must be obtuse hence tangent is negative $\tan\alpha = -\frac{8}{\sqrt{17}} = -\frac{8\sqrt{17}}{17}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Explains that equal gradients implies that lines are parallel	AO2.4	E1	Parallel lines have equal gradient
	Finds the gradient of the given line CAO	AO1.1b	B1	$2x + 3y + 4 = 0 \Rightarrow y = -\frac{2}{3}x - \frac{4}{3}$ So gradient is $-\frac{2}{3}$
	Finds the gradient of the line through the 2 given points CAO	AO1.1b	B1	Gradient of line through (9, 4) and (3, 8) is $\frac{8-4}{3-9} = -\frac{2}{3}$
	Deduces that the two lines are parallel	AO2.2a	R1	So line with equation $2x + 3y + 4 = 0$ is parallel to the line joining the points with coordinates (9, 4) and (3, 8) as both have gradient $-\frac{2}{3}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses binomial theorem to expand bracket – correct unsimplified expression but condone sign error	AO1.1a	M1	$1+\binom{10}{1}(-2x)^1+\binom{10}{2}(-2x)^2$
	Obtains constant term and x term, both correct	AO1.1b	A1	$=1-20x+180x^2...$
	Obtains correct x^2 term	AO1.1b	A1	
(b)	Selects x = 0.001	AO3.1a	B1	Substituting $x = 0.001$
	Substitutes 'their' chosen value of x into 'their' expansion from part **(a)** to obtain a 5 decimal place value	AO1.1a	M1	$1 - 0.020 + 0.000180 = 0.98018$
	Gives a correct explanation to confirm that the value found from the calculator is 0.98018 to 5 decimal places which is the same as the value found by using the expansion	AO2.4	A1	$0.998^{10} = 0.980179\ldots = 0.98018$ to 5 dp, which matches Carly's value.
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Substitutes $3 + h$ to obtain a correct unsimplified expression for $f(3 + h)$	AO1.1a	M1	$(3+h)^2-4(3+h)+2$ or $=9+6h+h^2-12-4h+2$
	Expresses simplified answer correctly in given format	AO1.1b	A1	$=h^2+2h-1$
(b)	Identifies and uses $\frac{f(x+h)-f(x)}{h}$ to obtain an expression for the gradient of chord Mark can be awarded for unsimplified expression.	AO1.1a	M1	Gradient of chord $=\frac{f(3+h)-f(3)}{h}$ $=\frac{h^2+2h-1+1}{h}$ $=h+2$ As $h\to 0$, $h+2\to 2$ Gradient of tangent $= 2$
	Obtains a correct and full simplification	AO1.1b	A1	
	Deduces that, as h approaches 0 the limit of $\frac{f(3+h)-f(3)}{h}$ is 2 (Must not simply say $h=0$ but accept words rather than limit notation) FT 'their' gradient provided M1 has been awarded	AO2.2a	R1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Obtains (at least four) correct $\log_{10} y$ values, in table or plotted	AO1.1a	M1	(1, 1.1) (2, 1.7) (3, 2.1) (4, 3.0) (5, 3.1) (6, 3.5)
	Plots **all** points correctly	AO1.1b	A1	(Points above plotted on grid)
(b)	Identifies $y = 1100$ and gives correct reason	AO2.2b	B1	(4, 1100), as it is not on the line that the other points are close to
(c)	Uses laws of logs. (May earn in part **(a)** if laws of logs were used there)	AO1.1a	M1	$\log_{10} y = \log_{10} k + x \log_{10} b$ Vertical intercept c = 0.68 (=$\log_{10} k$) Therefore from intercept: $k = 10^{0.68}$ Gradient m = 0.48 = $\log_{10} b$ Therefore from gradient: $b = 10^{0.48}$
	Draws straight line **and** calculates/measures the vertical intercept c and attempts 10^c **or** calculates/measures gradient m and attempts 10^m Alternatively uses regression line from calculator to get intercept and gradient	AO1.1a	M1	
	Finds correct value of b from 'their' gradient, provided 0.45< 'their' gradient < 0.51	AO1.1b	A1F	k = 4.8
	Finds correct value of k from 'their' intercept, provided 0.6 ≤ 'their' intercept ≤ 0.8	AO1.1b	A1F	b = 3.0
	Total		**7**	

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="7">11</td><td>Obtains $\frac{dy}{dx}$

for both the given curves – at least one term must be correct for each curve</td><td>AO3.1a</td><td>M1</td><td rowspan="7">$\frac{dy}{dx} = 6x^2 + 12x - 12$
$\frac{dy}{dx} = 60 - 12x$

Chris’s claim is incorrect when

$6x^2 + 12x - 12 \leq 60 - 12x$
$2x^2 + 8x - 24 \leq 0$
$x^2 + 4x - 12 \leq 0$
$(x + 6)(x - 2) \leq 0$

Critical values are $x =$ –6 and 2

<table><tr><td>region</td><td>$x < -6$</td><td>$-6 < x < 2$</td><td>$x > 2$</td></tr><tr><td>sign</td><td>+</td><td>–</td><td>+</td></tr></table>
$-6 \leq x \leq 2$

Chris’s claim is incorrect for values of x in the range $-6 \leq x \leq 2$, so he is wrong</td></tr>
<tr><td>States both derivatives correctly</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Translates problem into an inequality</td><td>AO3.1a</td><td>M1</td></tr>
<tr><td>States a correct quadratic inequality

FT from an incorrect $\frac{dy}{dx}$ provided both M1 marks have been awarded</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Determines a solution to ‘their’ inequality</td><td>AO1.1a</td><td>M1</td></tr>
<tr><td>Obtains correct range of values for x

Must be correctly written with both inequality signs correct</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Interprets final solution in context of the original question, must refer to Chris’s claim</td><td>AO3.2a</td><td>R1</td></tr>
<tr><td></td><td>Total</td><td></td><td>7</td><td></td></tr>
</table>

Q	Marking Instructions	AO	Marks	Typical Solution
12(a)	Rewrites given expression with a fractional power and negative power – at least one index form must be correct	AO1.1a	M1	$y = 6x^{\frac{3}{2}} + 32x^{-1}$
	Both terms correct	AO1.1b	A1	
	Differentiates 'their' rewritten expression – at least one term correct	AO1.1a	M1	$\frac{dy}{dx} = 6 \times \frac{3}{2} \times x^{\frac{1}{2}} - 32x^{-2}$ $= 9\sqrt{x} - \frac{32}{x^2}$
	Both terms correct for 'their' expression	AO1.1b	A1F	
(b)	Finds the equation of the tangent, a clear attempt must be seen	AO3.1a	M1	When $x = 4$,
	Evaluates 'their' $\frac{dy}{dx}$ (from part **(a)**) correctly (when $x = 4$)	AO1.1b	A1F	$\frac{dy}{dx} = 9 \times 2 - \frac{32}{16} = 16$ and
	Obtains correct y value (when $x = 4$)	AO1.1b	A1	$y = 6 \times 4 \times 2 + \frac{32}{4} = 56$
	Obtains correct form of the equation of a straight line using 'their' values for y and $\frac{dy}{dx}$	AO1.1b	A1F	Tangent: $y - 56 = 16(x - 4)$ When $y = 0$,
	Deduces value required at x-axis is when y equals 0 (follow through from 'their' equation) Both coordinates needed, any form	AO2.2a	A1F	$x = 4 - \frac{56}{16} = 0.5$ (0.5, 0)
	Total		**9**	

Section A

Answer **all** questions in the spaces provided.

1 $p(x) = x^3 - 5x^2 + 3x + a$, where a is a constant.

Given that $x - 3$ is a factor of p(x), find the value of a

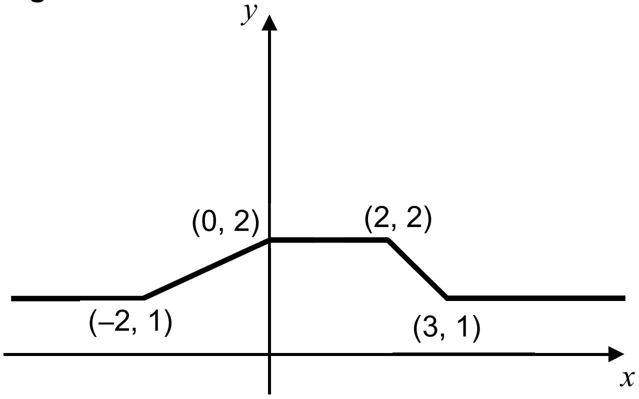
Circle your answer.

[1 mark]

–9 –3 3 9

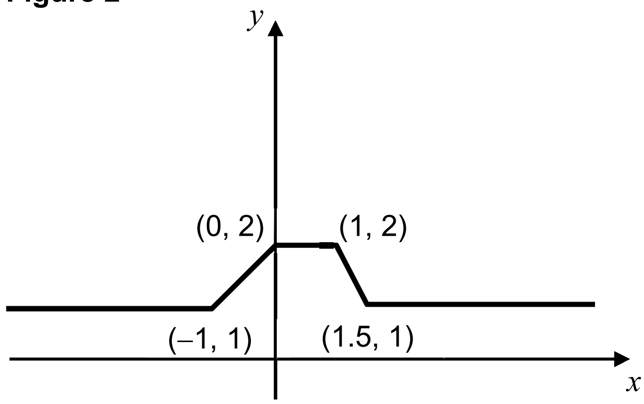
2 The graph of $y = \mathrm{f}(x)$ is shown in **Figure 1**.

Figure 1



State the equation of the graph shown in **Figure 2**.

Figure 2



Circle your answer.

[1 mark]

$y = \mathrm{f}(2x)$ $y = \mathrm{f}\left(\frac{x}{2}\right)$ $y = 2\mathrm{f}(x)$ $y = \frac{1}{2}\mathrm{f}(x)$

3 Find the value of $\log_a\left(a^3\right)+\log_a\left(\frac{1}{a}\right)$

[2 marks]

4 Find the coordinates, in terms of a, of the minimum point on the curve $y=x^2-5x+a$, where a is a constant.

Fully justify your answer.

[3 marks]

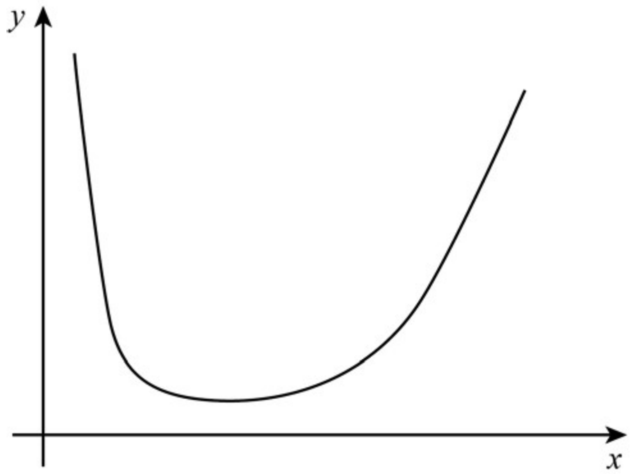
5 The quadratic equation $3x^2 + 4x + (2k - 1) = 0$ has real and distinct roots.

Find the possible values of the constant k

Fully justify your answer.

[4 marks]

6 A curve has equation $y = 6x^2 + \frac{8}{x^2}$ and is sketched below for $x > 0$



Find the area of the region bounded by the curve, the x-axis and the lines $x = a$ and $x = 2a$, where $a > 0$, giving your answer in terms of a

[4 marks]

7 Solve the equation

$$\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta \quad \text{where} \quad \cos\theta \neq 0$$

Give **all** values of θ to the nearest degree in the interval $0° < \theta < 180°$

Fully justify your answer.

[5 marks]

8 Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

9 A curve has equation $y = e^{2x}$

Find the coordinates of the point on the curve where the gradient of the curve is $\frac{1}{2}$

Give your answer in an exact form.

[5 marks]

10 David has been investigating the population of rabbits on an island during a three-year period.

Based on data that he has collected, David decides to model the population of rabbits, R, by the formula

$$R = 50e^{0.5t}$$

where t is the time in years after 1 January 2016.

10 (a) Using David's model:

10 (a) (i) state the population of rabbits on the island on 1 January 2016;

[1 mark]

10 (a) (ii) predict the population of rabbits on 1 January 2021.

[1 mark]

10 (b) Use David's model to find the value of t when $R = 150$, giving your answer to three significant figures.

[2 marks]

10 (c) Give **one** reason why David's model may **not** be appropriate.

[1 mark]

10 (d) On the same island, the population of crickets, C, can be modelled by the formula

$$C = 1000e^{0.1t}$$

where t is the time in years after 1 January 2016.

Using the two models, find the year during which the population of rabbits first exceeds the population of crickets.

[3 marks]

Q	Marking Instructions	AO	Marks	Typical solution
1	Circles correct answer	AO1.1b	B1	9
	Total		**1**	
2	Circles correct answer	AO1.2	B1	$y = \mathrm{f}(2x)$
	Total		**1**	
3	Correctly applies a single law of logs with either term	AO1.1a	M1	$\log_a(a^3) + \log_a\left(\frac{1}{a}\right) = 3 + (-1)$
	States correct final answer (NMS scores full marks)	AO1.1b	A1	$= 3 - 1$ $= 2$
	Total		**2**	
4	Selects an appropriate method – **either** differentiates, at least as far as: $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x....$ **or** commences completion of the square: $\left(x - \frac{5}{2}\right)^2 + ...$	AO1.1a	M1	$y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + a$ y minimised when squared bracket is 0 $\left(\frac{5}{2},\ a - \frac{25}{4}\right)$ **ALT** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 5$
	Fully differentiates **and** sets derivative equal to zero **or** fully completes square Allow one error	AO1.1a	M1	so $2x - 5 = 0$ for minimum $x = \frac{5}{2}$
	Obtains both coordinates	AO1.1b	A1	$y = \left(\frac{5}{2}\right)^2 - 5\left(\frac{5}{2}\right) + a = a - \frac{25}{4}$
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Forms discriminant – condone one error in discriminant	AO1.1a	M1	for distinct real roots, disc > 0 $4^2 - 4\times 3\times(2k-1) > 0$ $16 - 12(2k-1) > 0$ $28 - 24k > 0$ $k < \frac{7}{6}$
	States that discriminant > 0 for real and distinct roots	AO2.4	R1	
	Forms an inequality from 'their' discriminant	AO1.1a	M1	
	Solves inequality for k correctly Allow un-simplified equivalent fraction	AO1.1b	A1	
	Total		**4**	
6	States a correct integral expression (ignore limits at this stage)	AO1.1a	M1	Area = $\int_a^{2a}\left(6x^2 + \frac{8}{x^2}\right)\mathrm{d}x$
	Integrates at least one term correctly	AO1.1b	A1	$= \left[2x^3 - \frac{8}{x}\right]_a^{2a}$
	Substitutes $2a$ and a into 'their' integrated expression	AO1.1a	M1	$= (16a^3 - \frac{4}{a}) - (2a^3 - \frac{8}{a})$
	States correct final answer with terms collected FT correct substitution into incorrect integral provided both M1 marks awarded	AO1.1b	A1F	$= 14a^3 + \frac{4}{a}$
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Divides or multiplies by $\cos\theta$	AO3.1a	M1	$\frac{\sin\theta\tan\theta}{\cos\theta}+2\frac{\sin\theta}{\cos\theta}=3$
	Obtains correct quadratic	AO1.1b	A1	$\tan^2\theta+2\tan\theta-3=0$
	Applies a correct method to solve 'their' quadratic PI	AO1.1a	M1	$(\tan\theta+3)(\tan\theta-1)=0$ $\tan\theta=1$ or -3
	Finds two correct values of $\tan\theta$ from 'their' quadratic	AO1.1b	A1F	$\theta=45°$ or $108°$ **ALT** $\sin\theta\tan\theta\cos\theta+2\sin\theta\cos\theta=3\cos^2\theta$
	Obtains two correct answers CAO	AO1.1b	A1	$\sin^2\theta+2\sin\theta\cos\theta-3\cos^2\theta=0$ $(\sin\theta+3\cos\theta)(\sin\theta-\cos\theta)=0$ $\tan\theta=1$ or -3 $\theta=45°$ or $108°$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	Explains clearly that $f(x)$ is increasing $\Leftrightarrow f'(x) > 0$ (for all values of x) or Explains $\Rightarrow f(x)$ is increasing $f'(x) > 0$ for all values of x This may appear at any appropriate point in their argument	AO2.4	E1	For all x, $f'(x) > 0 \Rightarrow f(x)$ is an increasing function $f(x) = x^3 - 3x^2 + 15x - 1$ $\Rightarrow f'(x) = 3x^2 - 6x + 15$ $\Rightarrow f'(x) = 3(x-1)^2 + 12$ $\therefore f'(x)$ has a minimum value of 12 therefore $f'(x) > 0$ for all values of x **OR** for $f'(x)$, $b^2 - 4ac = -144$ $\therefore f'(x) \neq 0$ for any real x, so $f'(x)$ is either always positive or always negative. $f'(0) = 15$ therefore $f'(x) > 0$ for all values of x **OR** $f''(x) = 6x - 6$, which $= 0$ when $x = 1$ so min $f'(x)$ is $f'(1) = 12$ therefore $f'(x) > 0$ for all values of x Thus, since, $f'(x) > 0$ for all values of x it is proven that $f(x)$ is an increasing function.
	Differentiates – at least two correct terms	AO1.1a	M1	
	All terms correct	AO1.1b	A1	
	Attempts a correct method which could lead to $f'(x) > 0$	AO3.1a	M1	
	Correctly deduces $f'(x) > 0$ (for all values of x)	AO2.2a	A1	
	Writes a clear statement that links the steps in the argument together, the deduction about a positive gradient for all values of x proves that the given function is increasing for all values of x	AO2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9	States the correct gradient of the curve	AO1.2	B1	Grad of curve $= 2e^{2x}$
	Forms an equation using 'their' gradient of the curve and puts it equal to $\frac{1}{2}$	AO1.1a	M1	= grad of tangent so $2e^{2x} = \frac{1}{2}$
	Takes a log of each side of 'their' equation and uses law of logs to obtain equation in x	AO1.1a	M1	$e^{2x} = \frac{1}{4} \Rightarrow 2x = \ln\left(\frac{1}{4}\right)$
	Obtains a correct exact value for x	AO1.1b	A1	$\Rightarrow x = \frac{1}{2}\ln\left(\frac{1}{4}\right) = \ln\left(\frac{1}{2}\right) = -\ln 2$
	Substitutes 'their' value of x and obtains y value and hence the coordinates (follow through provided values are exact)	AO1.1b	A1F	$y = e^{2x} = \frac{1}{4}$ $\left(-\ln 2, \frac{1}{4}\right)$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)(i)	States correct value CAO	AO3.4	B1	50
(a)(ii)	States correct integer value CAO	AO3.4	B1	609
(b)	Forms correct equation and rearranges to obtain $e^{0.5t} = \ldots$	AO3.4	M1	$150 = 50e^{0.5t}$ so $e^{0.5t} = 3$
	Obtains the correct solution. Must give answer to 3 sf	AO1.1b	A1	$t = 2\ln 3 = 2.20$
(c)	1 mark for any clear valid reason, must be set in context of the question	AO3.5b	E1	No constraint on the number of rabbits (ie could go off to infinity) **OR** Model is only based on the 3 years of the study. Things may change **OR** Continuous model but number of rabbits is discrete **OR** Ignores extraneous factors such as disease, predation, limited food supply
(d)	Forms an equation with exponentials by letting $R = C$ PI	AO3.4	M1	$1000e^{0.1t} = 50e^{0.5t}$ $20 = e^{0.4t}$ $t = \ln 20 \div 0.4$ $= 7.49$ 2023
	Solves 'their' equation correctly	AO1.1a	M1	
	States correct answer as the year 2023 CAO NMS scores full marks for 2023	AO3.2a	A1	
	Total		**8**	

Answer **all** questions in the spaces provided.

1 Find the gradient of the line with equation $2x + 5y = 7$

Circle your answer.

[1 mark]

$\frac{2}{5}$ $\frac{5}{2}$ $-\frac{2}{5}$ $-\frac{5}{2}$

2 A curve has equation $y = \frac{2}{\sqrt{x}}$

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$

Circle your answer.

[1 mark]

$\frac{\sqrt{x}}{3}$ $\frac{1}{x\sqrt{x}}$ $-\frac{1}{x\sqrt{x}}$ $-\frac{1}{2x\sqrt{x}}$

3 When θ is small, find an approximation for $\cos 3\theta + \theta \sin 2\theta$, giving your answer in the form $a + b\theta^2$

[3 marks]

Turn over for the next question

4 $p(x) = 2x^3 + 7x^2 + 2x - 3$

4 (a) Use the factor theorem to prove that $x + 3$ is a factor of $p(x)$

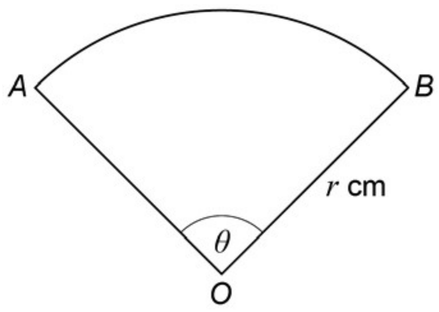
[2 marks]

4 (b) Simplify the expression $\dfrac{2x^3 + 7x^2 + 2x - 3}{4x^2 - 1}$, $x \neq \pm\dfrac{1}{2}$

[4 marks]

Turn over for the next question

5 The diagram shows a sector *AOB* of a circle with centre *O* and radius r cm.



The angle *AOB* is θ radians

The sector has area 9 cm^2 and perimeter 15 cm.

5 (a) Show that r satisfies the equation $2r^2 - 15r + 18 = 0$

[4 marks]

5 (b) Find the value of θ . Explain why it is the only possible value.

[4 marks]

Turn over for the next question

7 Find the values of k for which the equation $(2k-3)x^2 - kx + (k-1) = 0$ has equal roots.

[4 marks]

Turn over for the next question

8 (a) Given that $u = 2^x$, write down an expression for $\frac{du}{dx}$

[1 mark]

8 (b) Find the exact value of $\int_0^1 2^x \sqrt{3 + 2^x}\ dx$

Fully justify your answer.

[6 marks]

9 A curve has equation $y = \dfrac{2x+3}{4x^2+7}$

9 (a) (i) Find $\dfrac{dy}{dx}$

[2 marks]

9 (a) (ii) Hence show that y is increasing when $4x^2 + 12x - 7 < 0$

[4 marks]

9 (b) Find the values of x for which y is increasing.

[2 marks]

Turn over for the next question

10 The function f is defined by

$$f(x) = 4 + 3^{-x} \text{ , } x \in \mathbb{R}$$

10 (a) Using set notation, state the range of f

[2 marks]

10 (b) The inverse of f is f^{-1}

10 (b) (i) Using set notation, state the domain of f^{-1}

[1 mark]

10 (b) (ii) Find an expression for $f^{-1}(x)$

[3 marks]

10 (c) The function g is defined by

$$\mathrm{g}(x) = 5 - \sqrt{x} \ , \ (x \in \mathbb{R} : x > 0)$$

10 (c) (i) Find an expression for $\mathrm{gf}(x)$

[1 mark]

10 (c) (ii) Solve the equation $\mathrm{gf}(x) = 2$, giving your answer in an exact form.

[3 marks]

17 $f(x) = \sin x$

Using differentiation from first principles find the exact value of $f'\left(\frac{\pi}{6}\right)$

Fully justify your answer.

[6 marks]

END OF QUESTIONS

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	$-\frac{2}{5}$
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	$-\frac{1}{x\sqrt{x}}$
	Total		**1**	
3	Uses either $\cos x \approx 1 - \frac{1}{2}x^2$ or $\sin x \approx x$ (PI)	AO1.2	B1	$\cos 3\theta + \theta \sin 2\theta \approx 1 - \frac{(3\theta)^2}{2} + \theta(2\theta)$
	Substitutes 2θ and 3θ into 'their' expression	AO1.1a	M1	$\approx 1 - \frac{5}{2}\theta^2$
	Obtains correct answer	AO1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Demonstrates $\text{p}(-3)=0$	AO1.1b	B1	$\text{p}(-3)=2(-3)^3+7(-3)^2+2(-3)-3$ $=-54+63-6-3=0$ $\text{p}(-3)=0 \Rightarrow x+3$ is a factor
	Constructs rigorous mathematical proof (to achieve this mark, the student must clearly calculate and state that $\text{p}(-3)=0$ **and** clearly state that this implies that $x+3$ is a factor)	AO2.1	R1	
(b)	Factorises the numerator and denominator (this mark is achieved for any reasonable attempt at factorisation through the selection of an appropriate method, eg long division)	AO1.1a	M1	$\dfrac{(x+3)(2x^2+x-1)}{(2x+1)(2x-1)}$ $=\dfrac{(x+3)(2x-1)(x+1)}{(2x+1)(2x-1)}$ $=\dfrac{(x+3)(x+1)}{(2x+1)},\ x\neq\pm\dfrac{1}{2}$
	Finds second factor in numerator or fully factorises denominator (PI by complete factorisation)	AO1.1b	A1	
	Finds fully correct factorised expression (PI by complete factorisation)	AO1.1b	A1	
	Obtains a completely correct solution with restriction on domain stated	AO1.1b	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Recalls $A=\frac{1}{2}r^2\theta$ or $l=r\theta$ PI by use in equation	AO1.2	B1	Area of sector gives $\frac{1}{2}r^2\theta=9$, $\theta=\frac{18}{r^2}$
	Constructs two equations at least one correct	AO1.1a	M1	Perimeter of sector gives $2r+r\theta=15$
	Eliminates θ FT incorrect equations	AO1.1a	M1	$2r+\frac{18}{r}=15$
	Constructs a rigorous mathematical argument to show required result, clearly constructing two correct simultaneous equations and eliminating θ **AG**	AO2.1	R1	$2r^2+18=15r$ $2r^2-15r+18=0$ **(AG)**
(b)	Solves a quadratic equation and finds two values of θ	AO3.1a	M1	$r=\frac{3}{2}, r=6$ $r=6\Rightarrow\theta=\frac{1}{2}$ $r=\frac{3}{2}\Rightarrow\theta=8$ $8>2\pi \therefore \theta\neq 8$ so only one possible value of θ
	Finds two correct values of r	AO1.1b	B1	
	Finds both values of θ	AO1.1b	A1	
	Gives a valid reason for rejecting one of 'their' values	AO2.4	R1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Clearly states that equal roots $\Rightarrow b^2-4ac=0$	AO2.4	B1	$\therefore b^2-4ac=0$ for equal roots $k^2-4(2k-3)(k-1)=0$ $7k^2-20k+12=0$ $k=\frac{6}{7}, k=2$
	Forms quadratic expression in k (allow one error)	AO1.1a	M1	
	Obtains correct quadratic equation in k (PI by correct values for k)	AO1.1b	A1	
	Obtains correct values for k for 'their' quadratic equation	AO1.1b	A1F	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	States the correct derivative	AO1.1b	B1	$2^x \ln 2$
(b)	Selects an appropriate method for integrating, which could lead to a correct exact solution (this could be indicated by an attempt at a substitution or attempting to write the integrand in the form $f'(x)f(x)^n$)	AO3.1a	M1	Let $u = 2^x$ Then $\frac{du}{dx} = 2^x \ln 2$ And $\frac{1}{\ln 2}\frac{du}{dx} = 2^x$
	Correctly writes integrand in a form which can be integrated (condone missing or incorrect limits)	AO1.1b	A1	$I = \int (3+u)^{\frac{1}{2}} \frac{1}{\ln 2}\frac{du}{dx}\, dx$ $= \frac{1}{\ln 2}\int (3+u)^{\frac{1}{2}}\, du$
	Integrates 'their' expression (allow one error)	AO1.1a	M1	$= \frac{2}{3\ln 2}(3+u)^{\frac{3}{2}} + c$
	Substitutes correct limits corresponding to 'their' method	AO1.1a	M1	Sub limits: $\left[\frac{2}{3\ln 2}(3+u)^{\frac{3}{2}}\right]_1^2$
	Obtains correct value in an exact form	AO1.1b	A1	$\frac{2}{3} \times \frac{1}{\ln 2}\left(5\sqrt{5} - 8\right)$ **ALT (direct inspection)**
	Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips Substitution should be clearly stated in exact form and change of variable or solution by direct inspection should be achieved correctly with correct use of symbols and connecting language	AO2.1	R1	$\int 2^x \sqrt{3+2^x}\, dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2\sqrt{3+2^x}\, dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2(3+2^x)^{\frac{1}{2}}\, dx$ $= \frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}$ $\left[\frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}\right]_0^1$ $\frac{2}{3} \times \frac{1}{\ln 2}(5\sqrt{5} - 8)$
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)(i)	Selects an appropriate routine procedure; evidence of quotient rule or product rule	AO1.1a	M1	$\frac{dy}{dx} = \frac{2(4x^2+7)-8x(2x+3)}{(4x^2+7)^2}$
	Obtains correct derivative (no need for simplification)	AO1.1b	A1	
(a)(ii)	States clearly that $\frac{dy}{dx} > 0 \Rightarrow y$ is increasing	AO2.4	R1	y is increasing $\Leftrightarrow \frac{dy}{dx} > 0$ $\frac{2(4x^2+7)-8x(2x+3)}{(4x^2+7)^2} > 0$
	Forms inequality from 'their' $\frac{dy}{dx} > 0$	AO3.1a	B1	$(4x^2+7)^2 > 0$ for all x $\therefore 2(4x^2+7)-8x(2x+3) > 0$
	Deduces numerator must be positive	AO2.2a	R1	$8x^2+14-16x^2-24x > 0$ $4x^2+12x-7 < 0$ **(AG)**
	Considers denominator alone and sets out clear argument to justify given inequality **AG** Only award this mark if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	
(b)	Solves the correct quadratic inequality (accept evidence of factorising, completing the square, use of formula, or correct critical values stated)	AO1.1a	M1	$(2x+7)(2x-1)$ $x = -\frac{7}{2}, \frac{1}{2}$
	Obtains fully correct answer, given as an inequality or using set notation	AO1.1b	A1	$-\frac{7}{2} < x < \frac{1}{2}$ **Or** $x \in \left(-\frac{7}{2}, \frac{1}{2}\right)$ **Or** $x \in \left[x : -\frac{7}{2} < x < \frac{1}{2}\right]$
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Makes a deduction about the lower bound of the function (4)	AO2.2a	B1	
	Correctly states the range of f using set notation	AO2.5	B1	The range of f is the set $(x : x > 4, x \in \mathbb{R})$
(b)(i)	States correctly the set they gave in part **(a)**	AO1.2	B1F	$(x : x > 4, x \in \mathbb{R})$
(b)(ii)	Interchanges x and y at any stage	AO1.1a	M1	$y = 4 + 3^{-x}$ $x = 4 + 3^{-y}$ $3^{-y} = x - 4$ $-y = \log_3(x - 4)$ $f^{-1}(x) = -\log_3(x - 4)$
	Rearranges and takes logs	AO1.1a	M1	
	Obtains correct expression from completely correct working for $f^{-1}(x)$, notation correct throughout	AO1.1b	A1	
(c)(i)	Obtains $gf(x)$	AO1.1b	B1	$gf(x) = g(4 + 3^{-x})$ $= 5 - (4 + 3^{-x})^{0.5}$
(c)(ii)	Forms equation and rearranges using 'their' $gf(x) = 2$	AO1.1a	M1	$5 - (4 + 3^{-x})^{0.5} = 2$ $(4 + 3^{-x}) = 9$ $3^{-x} = 5$ $x = -\log_3 5$
	Correctly rearranges to get a single exponential term where logs can be taken. (Follow through provided 'their' equation requires the use of logs.)	AO1.1b	A1F	
	Obtains correct solution	AO1.1b	A1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
17	Translates $f'\left(\frac{\pi}{6}\right)$ into $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}$	AO1.1a	M1	$f'\left(\frac{\pi}{6}\right)=\lim_{h\to 0}\left[\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}\right]$
	Uses sin ($A+B$) identity to replace $\sin\left(\frac{\pi}{6}+h\right)$, to commence argument (at least two lines of argument seen)	AO2.1	M1	$=\lim_{h\to 0}\left[\frac{\sin\frac{\pi}{6}\cos h+\cos\frac{\pi}{6}\sin h-\sin\left(\frac{\pi}{6}\right)}{h}\right]$ $=\lim_{h\to 0}\left[\frac{\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h-\frac{1}{2}}{h}\right]$
	Obtains correct two term expression involving cos h and sin h	AO1.1b	A1	$=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{\cos h-1}{h}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$ $=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{-2\sin^2\left(\frac{h}{2}\right)}{\frac{2h}{2}}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$
	Deduce what happens as $h\to 0$, for one part of 'their' expression using the limit of $\frac{\sin h}{h}$ OR by using small angles approximations	AO2.2a	R1	$=\left(\lim_{h\to 0}\frac{-\sin\left(\frac{h}{2}\right)}{2}\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right)+\frac{\sqrt{3}}{2}\lim_{h\to 0}\frac{\sin h}{h}$
	Deduce what happens as $h\to 0$, for the second part of 'their' expression using the limit of (cos h – 1)h OR by using small angle approximations	AO2.2a	R1	$=0\times 1+\frac{\sqrt{3}}{2}\times 1$ $=\frac{\sqrt{3}}{2}$
	Completes a rigorous argument leading to the correct exact value, with all the steps in the method clearly shown.	AO2.1	R1	
	Total		**6**	
	TOTAL		**100**	

Section A

Answer **all** questions in the spaces provided.

1 State the values of $|x|$ for which the binomial expansion of $(3+2x)^{-4}$ is valid.

Circle your answer.

[1 mark]

$|x| < \frac{2}{3}$ $\quad$ $|x| < 1$ $\quad$ $|x| < \frac{3}{2}$ $\quad$ $|x| < 3$

2 A zoologist is investigating the growth of a population of red squirrels in a forest.

She uses the equation $N = \dfrac{200}{1+9e^{-\frac{t}{5}}}$ as a model to predict the number of squirrels, N, in the population t weeks after the start of the investigation.

What is the size of the squirrel population at the start of the investigation?

Circle your answer.

[1 mark]

5 $\quad$ 20 $\quad$ 40 $\quad$ 200

5 (a) Determine a sequence of transformations which maps the graph of $y = \cos\theta$ onto the graph of $y = 3\cos\theta + 3\sin\theta$

Fully justify your answer.

[6 marks]

5 (b) Hence or otherwise find the least value and greatest value of

$$4+\left(3\cos\theta+3\sin\theta\right)^2$$

Fully justify your answer.

[3 marks]

Turn over for the next question

6 A curve *C*, has equation $y = x^2 - 4x + k$, where k is a constant.

It crosses the x-axis at the points $\left(2+\sqrt{5}, 0\right)$ and $\left(2-\sqrt{5}, 0\right)$

6 (a) Find the value of k .

[2 marks]

6 (b) Sketch the curve *C*, labelling the exact values of all intersections with the axes.

[3 marks]

Turn over for the next question

8 A curve has equation $y = 2x\cos 3x + \left(3x^2 - 4\right)\sin 3x$

8 (a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer in the form $\left(mx^2 + n\right)\cos 3x$, where m and n are integers.

[4 marks]

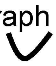
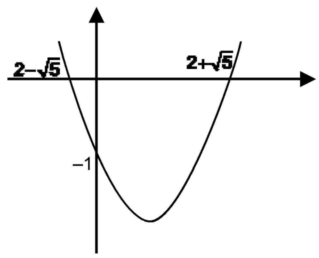
8 (b) Show that the x-coordinates of the points of inflection of the curve satisfy the equation

$$\cot 3x = \frac{9x^2 - 10}{6x}$$

[4 marks]

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	$\lvert x \rvert < \frac{3}{2}$
	Total		**1**	
2	Circles correct answer	AO3.4	B1	20
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Compares with $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$	AO3.1a	M1	$R\cos(\theta - \alpha)$ $\equiv R\sin\alpha\cos\theta + R\sin\theta\sin\alpha$
	Identifies version which will allow them to solve the problem	AO3.1a	A1	$\therefore R\cos\alpha = 3$ and $R\sin\alpha = 3$ $R = \sqrt{18}$
	Obtains correct R	AO1.1b	A1	$\alpha = \frac{\pi}{4}$
	Obtains correct α	AO1.1b	A1	$\therefore 3\cos\theta + 3\sin\theta \equiv \sqrt{18}\cos\left(\theta - \frac{\pi}{4}\right)$
	Interprets 'their' equation to identify first transformation	AO3.2a	E1	Which is a stretch in the y-direction scale factor $\sqrt{18}$
	Interprets 'their' equation to identify second transformation	AO3.2a	E1	and a translation $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$
(b)	Constructs a rigorous mathematical argument, to find either the least or greatest value Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips (no FT for this mark)	AO2.1	R1	$4 + (3\cos\theta + 3\sin\theta)^2$ $4 + \left(\sqrt{18}\cos\left(\theta + \frac{\pi}{4}\right)\right)^2$ Least value occurs when $\cos^2\left(\theta + \frac{\pi}{4}\right) = 0$ $\therefore$least value = 4
	Deduces the least value Using 'their' values of R and α	AO2.2a	A1F	Greatest value occurs when $\cos^2\left(\theta + \frac{\pi}{4}\right) = 1$
	Deduces the greatest value Using 'their' values of R and α	AO2.2a	A1F	greatest value $= 4 + 18$ $= 22$
	Total		**9**	

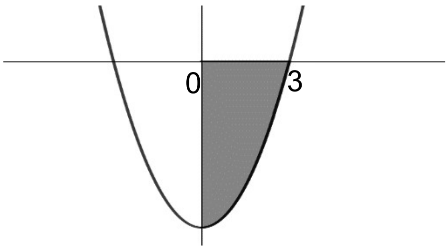
Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Uses either of the given coordinates in the given equation (accept product of the roots)	AO1.1a	M1	$k = 4(2+\sqrt{5}) - (2+\sqrt{5})^2 = -1$ **ALT** $k = 4(2-\sqrt{5}) - (2-\sqrt{5})^2 = -1$ **ALT** $k = (2-\sqrt{5})(2+\sqrt{5}) = -1$
	Obtains the correct value of k	AO1.1b	A1	
6(b)	Sketches a graph with the correct shape 	AO1.2	B1	
	Deduces correct relative positioning of intersections with axes (must see labels)	AO2.2a	B1	
	Deduces minimum lies to right of y-axis in fourth quadrant	AO2.2a	B1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Uses the product rule for either term	AO1.1a	M1	$y = 2x\cos 3x + (3x^2 - 4)\sin 3x$
	Uses the product rule for both terms	AO1.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 3x - 6x\sin 3x$ $+6x\sin 3x + 3(3x^2 - 4)\cos 3x$
	Differentiates both terms correctly	AO1.1b	A1	$= (9x^2 - 10)\cos 3x$
	Rearranges to correct form CAO	AO1.1b	A1	
(b)	Finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ from 'their' first derivative and equates to zero	AO3.1a	M1	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 18x\cos 3x - 3(9x^2 - 10)\sin 3x$
	Applies product rule correctly on 'their' $\frac{\mathrm{d}y}{\mathrm{d}x}$ FT only applies if both M1 marks awarded in part **(a)**	AO1.1b	A1F	point of inflection $\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0 \Rightarrow$ $18x\cos 3x - 3(9x^2 - 10)\sin 3x = 0$ $\Rightarrow \frac{\cos 3x}{\sin 3x} = \frac{3(9x^2 - 10)}{18x}$
	Arrives at a result using 'their' second derivative through correct algebraic manipulation that is correct for 'their' second derivative FT only applies if both previous marks in **(b)** have been awarded.	AO1.1b	A1F	$\Rightarrow \cot 3x = \frac{9x^2 - 10}{6x}$ **(AG)**
	Constructs a clearly explained rigorous mathematical argument, to show the required result This must include a concluding statement or an explanation of reasoning at the start. **AG**	AO2.1	R1	
	Total		**8**	

Section A

Answer **all** questions in the spaces provided.

1 The graph of $y = x^2 - 9$ is shown below.

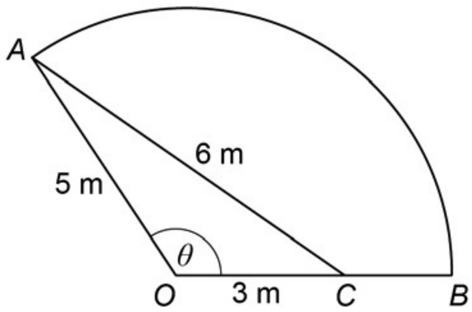


Find the area of the shaded region.
Circle your answer.

[1 mark]

–18 –6 6 18

2 A wooden frame is to be made to support some garden decking. The frame is to be in the shape of a sector of a circle. The sector *OAB* is shown in the diagram, with a wooden plank *AC* added to the frame for strength. *OA* makes an angle of θ with *OB*.



2 (a) Show that the exact value of $\sin\theta$ is $\frac{4\sqrt{14}}{15}$

[3 marks]

2 (b) Write down the value of θ in radians to 3 significant figures.

[1 mark]

2 (c) Find the area of the garden that will be covered by the decking.

[2 marks]

4 $\int_1^2 x^3 \ln(2x)\,dx$ can be written in the form $p\ln 2 + q$, where p and q are rational numbers.

Find p and q.

[5 marks]

5 (a) Find the first three terms, in ascending powers of x , in the binomial expansion of $(1+6x)^{\frac{1}{3}}$

[2 marks]

5 (b) Use the result from part **(a)** to obtain an approximation to $\sqrt[3]{1.18}$ giving your answer to 4 decimal places.

[2 marks]

5 (c) Explain why substituting $x=\frac{1}{2}$ into your answer to part **(a)** does not lead to a valid approximation for $\sqrt[3]{4}$.

[1 mark]

6 Find the value of $\int_1^2 \frac{6x+1}{6x^2-7x+2}\,\mathrm{d}x$, expressing your answer in the form

$m\ln 2 + n\ln 3$, where m and n are integers.

[8 marks]

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	18
	Total		**1**	
2(a)	Makes clear attempt to use the cosine rule	AO3.1a	M1	$6^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos\theta$
	Uses trig identity with 'their' cos θ	AO1.1a	M1	$\cos\theta = \frac{3^2+5^2-6^2}{30} = -\frac{1}{15}$
	Constructs rigorous argument leading to correct result **AG** Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	$\therefore \sin\theta = \sqrt{1-\left(-\frac{1}{15}\right)^2}$ $\sin\theta = \frac{4\sqrt{14}}{15}$ (**AG**)
(b)	Writes down correct angle	AO2.2a	B1	1.64
(c)	Uses 'their' angle in $\frac{1}{2}r^2\theta$	AO1.1a	M1	$A = \frac{1}{2} \times 5^2 \times 1.64$
	Correct area FT use of incorrect obtuse angle provided both M1 marks awarded in part **(a)** and M1 awarded in **(c)**	AO1.1b	A1F	$= 20.5$ m^2
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Selects a method of integration, which could lead to a correct solution. Evidence of integration by parts OR an attempt at integration by inspection.	AO3.1a	M1	$u=\ln 2x;\quad \frac{dv}{dx}=x^3$ $\frac{du}{dx}=\frac{1}{x};\quad v=\frac{x^4}{4}$ $\left[\frac{x^4}{4}\ln(2x)\right]_1^2-\int_1^2\frac{x^3}{4}\,dx$ $\left[\frac{x^4}{4}\ln(2x)-\frac{x^4}{16}\right]_1^2$
	Applies integration by parts formula correctly OR correctly differentiates an expression of the form $Ax^4\ln 2x$	AO1.1b	A1	$=\left(\frac{2^4}{4}\ln(4)-\frac{2^4}{16}\right)-\left(\frac{1}{4}\ln(2)-\frac{1}{16}\right)$ $\frac{31}{4}\ln 2-\frac{15}{16}$
	Obtains correct integral, condone missing limits.	AO1.1b	A1	so $p=\frac{31}{4}\qquad q=-\frac{15}{16}$
	Substitutes correct limits into 'their' integral	AO1.1a	M1	**ALT** $\frac{d}{dx}(x^4\ln 2x)=4x^3\ln 2x+x^4.\frac{1}{x}$
	Obtains correct p and q FT use of incorrect integral provided both M1 marks have been awarded	AO1.1b	A1F	$\therefore\int_1^2 x^3\ln 2x\,dx=\left[\frac{1}{4}(x^4\ln 2x-\frac{x^4}{4})\right]_1^2$ $=\left(\frac{2^4}{4}\ln(4)-\frac{2^4}{16}\right)-\left(\frac{1}{4}\ln(2)-\frac{1}{16}\right)$ $\frac{31}{4}\ln 2-\frac{15}{16}$ $p=\frac{31}{4}\qquad q=-\frac{15}{16}$
	Total		**5**	

	Marking Instructions	AO	Marks	Typical Solution
5(a)	Uses binomial expansion, with at least two terms correct, may be un-simplified	AO1.1a	M1	$(1+6x)^{\frac{1}{3}} \approx 1+\frac{1}{3}\cdot 6x+\frac{1}{3}\cdot\frac{-2}{3}\cdot\frac{(6x)^2}{2}$
	Obtains correct simplified answer	AO1.1b	A1	$(1+6x)^{\frac{1}{3}} \approx 1+2x-4x^2$
(b)	Determines the correct value for x and substitutes this into 'their' answer to part **(a)**	AO3.1a	M1	$x=0.03$
	Obtains correct approximation for 'their' answer to part **(a)** FT allowed only if M1 from part **(a)** and M1 from part **(b)** have been awarded	AO1.1b	A1F	$\sqrt[3]{1.18} \approx 1 + 2(0.03) - 4(0.03)^2$ ≈ 1.0564
(c)	Explains the limitation of the expansion found in part **(a)** with reference to $x=\frac{1}{2}$	AO2.4	E1	Although $\left(1+6\times\frac{1}{2}\right)^{\frac{1}{3}}=\sqrt[3]{4}$ $x=\frac{1}{2}$ cannot be used since the expansion is only valid for $\lvert x\rvert<\frac{1}{6}$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Uses partial fractions with linear denominators $\frac{6x+1}{6x^2-7x+2} = \frac{A}{ax+b} + \frac{B}{cx+d}$	AO3.1a	M1	$\frac{6x+1}{6x^2-7x+2} = \frac{A}{3x-2} + \frac{B}{2x-1}$ $A(2x-1)+B(3x-2) = 6x+1$
	Obtains correct linear denominators	AO1.1b	B1	$x = \frac{2}{3},\ A\left(\frac{1}{3}\right) = 5$ so $A = 15$
	Obtains at least one numerator correct (using any valid method, eg equating coefficients or substitution of values)	AO1.1b	A1	$x = \frac{1}{2},\quad B\left(-\frac{1}{2}\right) = 4$ so $B = -8$ $\int_1^2 \frac{15}{3x-2} - \frac{8}{2x-1}\,dx$ $= \left[5\ln(3x-2) - 4\ln(2x-1)\right]_1^2$
	Obtains partial fractions completely correct	AO1.1b	A1	$= 5\ln(4) - 4\ln(3) - (5\ln(1)4\ln(1))$ $= 10\ln(2) - 4\ln(3)$
	Integrates 'their' partial fractions, must include logs $p\ln(ax+b) + q\ln(cx+d)$	AO1.1a	M1	
	'Their' integral correct (ignore limits)	AO1.1b	A1F	
	Substitutes limits into 'their' integral	AO1.1a	M1	
	Correct final answer in correct form **CAO**	AO1.1b	A1	
	Total		**8**	