7 Appendix B: mathematical formulae and identities

Students must be able to use the following formulae and identities for AS and A-level further mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure mathematics

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a(xy)$

$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$

$k\log_a x \equiv \log_a\left(x^k\right)$

Coordinate geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of an arithmetic progression: $u_n = a + (n - 1)d$

General term of a geometric progression: $u_n = ar^{n-1}$

Trigonometry

In the triangle *ABC*

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area = $\frac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2\sin A\cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d :

$C = 2\pi r = \pi d$

$A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a , b and c are the lengths of the sides and c is the hypotenuse:

$c^2 = a^2 + b^2$

Area of a trapezium = $\frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius r , where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$s = r\theta$

$A = \frac{1}{2}r^2\theta$

Complex numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 + \theta_2)}$

Loci in the Argand diagram:

$|z - a| = r$ is a circle radius r centred at a

$\arg(z - a) = \theta$ is a half line drawn from a at angle θ to a line parallel to the positive real axis.

Exponential form:

$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

the inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix **AB** is the transformation represented by matrix **B** followed by the transformation represented by matrix **A**.

For matrices **A**, **B**:

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Algebra

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

For $ax^2 + bx + c = 0$ with roots α and β :

$\alpha + \beta = \frac{-b}{a}$

$\alpha\beta = \frac{c}{a}$

For $ax^3 + bx^2 + cx + d = 0$ with roots α , β and γ :

$\sum\alpha = \frac{-b}{a}$

$\sum\alpha\beta = \frac{c}{a}$

$\alpha\beta\gamma = \frac{-d}{a}$

Hyperbolic functions

$\cosh x \equiv \frac{1}{2}(e^{x} + e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^{x} - e^{-x})$

$\tanh x \equiv \frac{\sinh x}{\cosh x}$

Calculus and differential equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c, x \neq 0$
$f'(x) + g'(x)$	$f(x) + g(x) + c$
$f'(g(x))g'(x)$	$f(g(x)) + c$

Area under a curve = $\int_a^b y \ \mathrm{d}x \ (y \geq 0)$

Volumes of revolution about the x and y axes:

$V_x = \pi\int_a^b y^2 \mathrm{d}x$

$V_y = \pi\int_c^d x^2 \mathrm{d}y$

Simple Harmonic Motion:

$\ddot{x} = -\omega^2 x$

Vectors

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

where θ is the acute angle between the vectors **a** and **b**.

The equation of the line through the point with position vector **a** parallel to vector **b** is:

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is:

$(\mathbf{r} - \mathbf{a}) . \mathbf{n} = 0$

Mechanics

Forces and equilibrium

Weight = mass × g

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$v = \frac{\mathrm{d}r}{\mathrm{d}t}$

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2r}{\mathrm{d}t^2}$

$r = \int v \ \mathrm{d}t$

$v = \int a \ \mathrm{d}t$

Statistics

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{X - \mu}{\sigma}$ where $X \sim \mathrm{N}(\mu, \sigma^2)$