6 Appendix A: mathematical notation

The tables below set out the notation that must be used by AS and A-level mathematics and further mathematics specifications. Students will be expected to understand this notation without need for further explanation.

Mathematics students will not be expected to understand notation that relates only to further mathematics content. Further mathematics students will be expected to understand all notation in the list.

For further mathematics, the notation for the core content is listed under sub headings indicating 'further mathematics only'. In this subject, awarding organisations are required to include, in their specifications, content that is additional to the core content. They will therefore need to add to the notation list accordingly.

AS students will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A-level content.

6.1 Set notation

1	Set notation	Meaning
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x: \ \ldots\}$	the set of all x such that …
1.7	$n(A)$	the number of elements in set A
1.8	$\varnothing$	the empty set
1.9	ε	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers {1, 2, 3, …}
1.12	$\mathbb{Z}$	the set of integers {0, ± 1, ± 2, ± 3, …}

1	Set notation	Meaning
1.13	$\mathbb{Z}^+$	the set of positive integers {1,2, 3, …}
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers {0, 1, 2, 3, …}
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers $\left\{\frac{p}{q}: p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$

Set notation (Further Maths only)

1	Set notation	Meaning
1.24	$\mathbb{C}$	the set of complex numbers

6.2 Miscellaneous symbols

2	Miscellaneous symbols	Meaning
2.1	=	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or is congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to

2	Miscellaneous symbols	Meaning
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant$, $\leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant$, $\geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term of an arithmetic or geometric sequence
2.17	l	last term of an arithmetic sequence
2.18	d	common difference of an arithmetic sequence
2.19	r	common ratio of a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence

Miscellaneous symbols (Further Maths only)

2	Miscellaneous symbols	Meaning
2.22	$\cong$	is isomorphic to

6.3 Operations

3	Operations	Meaning
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b

3	Operations	Meaning
3.3	$a \times b$, ab , $a \,.\, b$	a multiplied by b
3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1, n \in \mathbb{N}; 0! = 1$
3.10	$\binom{n}{r}$, ${}^{n}\mathrm{C}_{r}$, ${}_{n}\mathrm{C}_{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for n , $r \in \mathbb{Z}_0^+$, $r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$, $r \in \mathbb{Z}_0^+$

Operations (Further Maths only)

3	Operations	Meaning
3.11	$a \times_n b$	multiplication modulo n of a by b
3.12	$a +_n b$	addition modulo n of a and b
3.13	$G = (\langle n \rangle, *)$	n is the generator of a given group G under the operation *

6.4 Functions

4	Functions	Meaning
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f}: x \mapsto y$	the function f maps the element x to the element y

4	Functions	Meaning
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
4.9	$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x}, \ddot{x}, \ldots$	the first, second, ... derivatives of x with respect to t
4.11	$\int y \ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y \ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$

6.5 Exponential and logarithmic functions

5	Exponential and logarithmic functions	Meaning
5.1	e	base of natural logarithms
5.2	$\mathrm{e}^x, \ \exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_{\mathrm{e}} x$	natural logarithm of x

6.6 Trigonometric functions

6	Trigonometric functions	Meaning
6.1	sin, cos, tan, cosec, sec, cot	the trigonometric functions
6.2	$\sin^{-1}, \cos^{-1}, \tan^{-1}$ arcsin, arccos, arctan	the inverse trigonometric functions
6.3	°	degrees
6.4	rad	radians

Trigonometric functions (Further Maths only)

6	Trigonometric functions	Meaning
6.5	$\text{cosec}^{-1}, \sec^{-1}, \cot^{-1}$, arccosec, arcsec, arccot	the inverse trigonometric functions
6.6	sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
6.7	$\sinh^{-1}, \cosh^{-1}, \tanh^{-1}$ $\text{cosech}^{-1}, \text{sech}^{-1}, \coth^{-1}$ arcsinh, arccosh, arctanh, arccosech, arcsech, arccoth	the inverse hyperbolic functions

6.7 Complex numbers (Further Maths only)

7	Complex numbers	Meaning
7.1	i , j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$r(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$

7	Complex numbers	Meaning
7.5	Re(z)	the real part of z, Re(z) = x
7.6	Im(z)	the imaginary part of z, Im(z) = y
7.7	$\lvert z \rvert$	the modulus of z , $\lvert z \rvert = r = \sqrt{x^2 + y^2}$
7.8	arg(z)	the argument of z , arg(z) = θ, $-\pi < \theta \leq \pi$
7.9	z^*	the complex conjugate of z , $x - \mathrm{i}y$

Matrices (Further Maths only)

8	Matrices	Meaning
8.1	**M**	a matrix **M**
8.2	**0**	zero matrix
8.3	I	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix **M**
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
8.6	Δ, det **M** or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix **M**
8.7	**Mr**	image of column vector **r** under the transformation associated with the matrix **M**

6.9 Vectors

9	Vectors	Meaning
9.1	**a** , $\underline{a}$, $\underset{\sim}{a}$	the vector **a** , $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment *AB*
9.3	$\hat{a}$	a unit vector in the direction of **a**
9.4	**i**, **j**, **k**	unit vectors in the directions of the cartesian coordinate axes

9	Vectors	Meaning
9.5	$\|\mathbf{a}\|, a$	the magnitude of **a**
9.6	$\|\overrightarrow{AB}\|, AB$	the magnitude of $\overrightarrow{AB}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}, a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	**r**	position vector
9.9	**s**	displacement vector
9.10	**v**	velocity vector
9.11	**a**	acceleration vector

Vectors (Further Maths only)

9	Vectors	Meaning
9.12	**a** . **b**	the scalar product of **a** and **b**

6.10 Differential equations (Further Maths only)

10	Differential equations	Meaning
10.1	ω	angular speed

6.11 Probability and statistics

11	Probability and statistics	Meaning
11.1	A, B, C etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B

11	Probability and statistics	Meaning
11.7	X, Y, R etc.	random variables
11.8	x, y, r etc.	values of the random variables X, Y, R etc.
11.9	$x_1, x_2, \ldots$	values of observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x), \mathrm{P}(X = x)$	probability function of the discrete random variable X
11.12	$p_1, p_{2, \ldots}$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	~	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	Z~ N (0,1)	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution N(0,1)
11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation

11	Probability and statistics	Meaning
11.28	H_0	null hypothesis
11.29	H_1	alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population

6.12 Mechanics

12	Mechanics	Meaning
12.1	kg	kilogram
12.2	m	metre
12.3	km	kilometre
12.4	m/s, m s^{-1}	metre(s) per second (velocity)
12.5	m/s^2 , m s^{-2}	metre(s) per second per second (acceleration)
12.6	F	Force or resultant force
12.7	N	newton
12.8	Nm	newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient of friction