THE COLLEGES OF OXFORD UNIVERSITY

PHYSICS

Wednesday 2 November 2011

Time allowed: 2 hours

*For candidates applying for Physics, and Physics and Philosophy*

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**There are two parts (A and B) to this test, carrying equal weight.**

Answers should be written on the question sheet in the spaces provided and you should attempt as many questions as you can from each part.

Marks for each question are indicated in the right hand margin. There are a total of 100 marks available and total marks for each section are indicated at the start of a section. You are advised to divide your time according to the marks available, and to spend equal effort on parts A and B.

**No calculators, tables or formula sheets may be used.**

Answers in Part A should be given exactly unless indicated otherwise. Numeric answers in Part B should be calculated to 2 significant figures.

Use $g = 10\,\mathrm{m\,s^{-2}}$.

**Do NOT turn over until told that you may do so.**

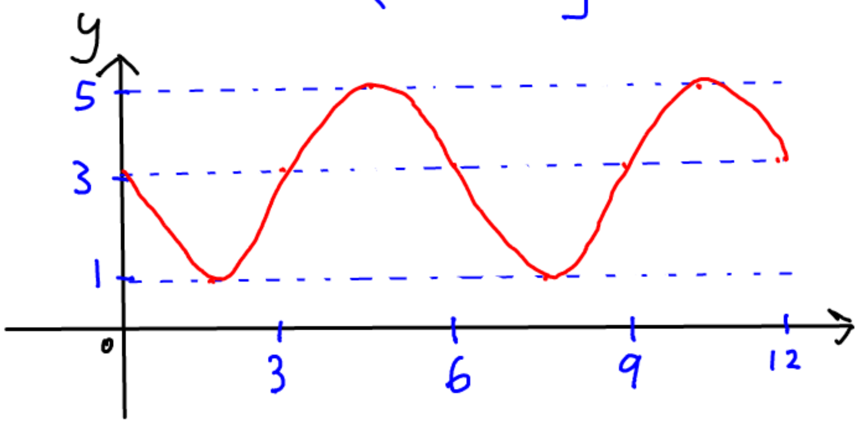
## Part A: Mathematics for Physics [50 Marks]

1. Find the values of $\theta$ between 0 and $2\pi$ which solve: $\sin\theta - 2\cos^2\theta = -1$. [4]

$$\sin\theta - 2(1-\sin^2\theta)+1=0$$
$$2\sin^2\theta+\sin\theta-1=0$$
$$(2\sin\theta-1)(\sin\theta+1)=0$$
$$\sin\theta=\frac{1}{2} \quad \text{or} \quad \sin\theta=-1$$
$$\theta=\frac{\pi}{6},\ \frac{5\pi}{6} \qquad \theta=\frac{3\pi}{2}$$

2. Sketch the function $y = 3 + 2\sin\left((x-3)\frac{\pi}{3}\right)$ between $x = 0$ and $x = 12$. [3]

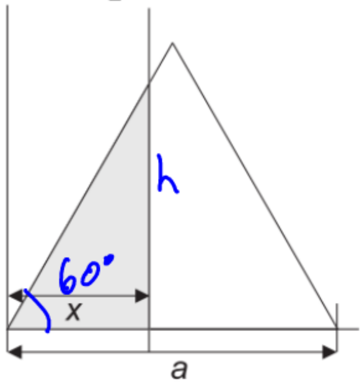
$$y=3+2\sin\left(\frac{\pi x}{3}-\pi\right)$$



3. The area $A$ is defined as the area to the left of point $x$, within an equilateral triangle with sides of length $a$. Find an expression for $A$ as a function of $x$,

(i) for $0 \leq x \leq \frac{a}{2}$

(ii) for $\frac{a}{2} \leq x \leq a$




[5]

i) $\tan 60 = \frac{h}{x}$

$h = \sqrt{3}\, x$

$A = \frac{1}{2} x h = \frac{\sqrt{3}}{2} x^2 \; ; \; 0 \leq x \leq \frac{a}{2}$

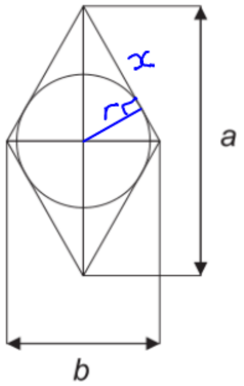
ii) When $x = \frac{a}{2}$, $h = \frac{\sqrt{3}}{2} a$

Area of whole triangle $= \frac{1}{2} a \frac{\sqrt{3}}{2} a$

$= \frac{\sqrt{3}}{4} a^2$

$\Rightarrow A = \frac{\sqrt{3}}{4} a^2 - \frac{\sqrt{3}}{2} (a-x)^2 \; ; \; \frac{a}{2} \leq x \leq a$

4. The figure below shows a rhombus with diagonals $a$ and $b$, which contains a circle. Find an expression for the ratio of the area of the circle to the area of the rhombus in terms of $a$ and $b$.




[6]

Using similar triangles,

$$\frac{b/2}{r} = \frac{x}{a/2}$$

$$r = \frac{ab}{4} \cdot \frac{1}{x} = \frac{ab}{4} \cdot \frac{1}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}}$$

$$\Rightarrow A_c = \pi r^2 = \pi\left(\frac{ab}{4}\right)^2 \cdot \frac{4}{a^2+b^2} = \frac{\pi a^2 b^2}{4(a^2+b^2)}$$

$$A_R = \frac{1}{2}ab$$

$$\Rightarrow \frac{A_c}{A_R} = \frac{\pi a^2 b^2}{4(a^2+b^2)} \cdot \frac{2}{ab}$$

$$= \frac{\pi ab}{2(a^2+b^2)}$$

5. Given that $\log 5 = 0.7$ (to one decimal place). Find the value of $x$ such that $2^x = 10$ (again to one decimal place). [4]

$$2^x = 10$$

$$x \log 2 = \log 10$$

$$x = \frac{\log 10}{\log 2} = \frac{\log 10}{\log 10 - \log 5} = \frac{\log 10}{\log 10 - 0.7}$$

$$= \frac{1}{1-0.7} = \frac{1}{0.3} = 3.3$$

6. Evaluate: $\sum_{r=1}^{6} \left(2^r + \frac{2r}{3}\right)$ [4]

$$\sum_{r=1}^{6} 2^r + \frac{2}{3}\sum_{r=1}^{6} r = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

$$+ \frac{2}{3}(1+2+3+4+5+6)$$

$$= 2+4+8+16+32+64+\frac{2}{3}(21)$$

$$= 126 + 14$$

$$= 140$$

7. Given that $x^2 - x - 6$ is a factor of $x^4 + 4x^3 - 17x^2 - 24x + 36 = 0$, find all the roots of this polynomial. [4]

$$0 = x^4 + 4x^3 - 17x^2 - 24x + 36 = (x^2 - x - 6)(x^2 + 5x - 6)$$

$$0 = (x-3)(x+2)(x+6)(x-1)$$

$$x = -6, -2, 1, 3$$

8. Evaluate the following integrals:

(i) $\int \frac{x+2}{(x+1)(x-1)}dx$ [3]

$$\int\left(\frac{1/-2}{x+1}+\frac{3/2}{x-1}\right)dx = \frac{3}{2}\int\frac{1}{x-1}dx - \frac{1}{2}\int\frac{1}{x+1}dx$$

$$= \frac{3}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + \ln A$$

$$= \ln\left[\frac{A(x-1)^{3/2}}{x+1}\right]$$

(ii) $\int_0^1 \frac{1}{\sqrt{x+1}}dx$ [3]

$$\int_0^1 (x+1)^{-1/2}dx = 2\left[(x+1)^{1/2}\right]_0^1$$

$$= 2(\sqrt{2}-1)$$

9. Given the functions:

$$y_1 = x^3 - 3x^2 + 2x + 3$$
$$y_2 = x^2 - 3x - 4$$

Find the values of $x$ between 0.8 and 1.9 which give the maximum and minimum difference between $y_1$ and $y_2$. [6]

$$\Delta = y_1 - y_2 = x^3 - 3x^2 + 2x + 3 - x^2 + 3x + 4$$
$$= x^3 - 4x^2 + 5x + 7$$
$$\frac{d\Delta}{dx} = 3x^2 - 8x + 5 = 0$$
$$(3x-5)(x-1) = 0$$
$$x = \frac{5}{3} \text{ or } 1$$

$$\frac{d^2\Delta}{dx^2} = 6x - 8 \quad \text{when } x = 1,\ \frac{d^2\Delta}{dx^2} = -2 < 0$$
$$x = \frac{5}{3},\ \frac{d^2\Delta}{dx^2} = 2 > 0$$

$\therefore$ $x = 1$ gives maximum, $x = \frac{5}{3}$ gives minimum

10. Given that $s = x^2 + y^2$ and $t = 2xy$, find expressions for $x$ and $y$ in terms of $s$ and $t$. [4]

$$x = \frac{t}{2y}$$

$$y = \frac{t}{2x} \Rightarrow s = x^2 + \left(\frac{t}{2x}\right)^2$$
$$0 = 4x^4 - 4sx^2 + t^2$$
$$x^2 = \frac{4s \pm \sqrt{16s^2 - 16t^2}}{8} = \frac{s \pm \sqrt{s^2 - t^2}}{2}$$

$$x = \pm\sqrt{\frac{s \pm (s^2 - t^2)^{1/2}}{2}}$$

$$y = \frac{t}{2x} = \pm\frac{t}{2}\sqrt{\frac{2}{s \pm (s^2 - t^2)^{1/2}}}$$

11. The numbers shown by two dice are labelled $d_1$ and $d_2$; a score is constructed from these by the expression: $S = Ad_1 + Bd_2 + C$, where $A$, $B$ and $C$ are constants. Determine the values of $A$, $B$ and $C$ such that the range of possible values for $S$ covers all integers from 0 to 35, with an equal probability of each score. [4]

Max = 35 $\Rightarrow$ when $d_1 = d_2 = 6$, $6A + 6B + C = 35$

Min = 0 $\Rightarrow$ when $d_1 = d_2 = 1$, $A + B + C = 0$

$\Rightarrow 6(A+B) = 35 - C$

$-6C = 35 - C$

$C = -7 \Rightarrow A + B = 7$

$\therefore A = 1$ or $6$ $\quad B = 6$ or $1$ $\quad C = -7$

# Part B: Physics [50 Marks]

## Multiple choice (10 marks)

Please circle **one** answer to each question only.

12. A boy sitting on a harbour wall observes waves on the water's surface. He sees that the waves have a period of 2 s and that a single wave travels the length of the harbour wall in 25 s. If the harbour wall is of length 45 m, what is the wavelength of the wave?

$\lambda = \frac{v}{f}$ $= \frac{45}{25} \times 2$ $= 3.6$ m

$v = \frac{45}{25}$, $f = \frac{1}{2}$

**A** 4.0 m　　**B** 1.8 m
**C** 3.6 m　　**D** 1.0 m　　[1]

13. The contents of a refrigerator, which are kept at a temperature $T = 6°C$, has to be cooled at a rate of $\alpha(T_S - T)$, where $T_S$ is the temperature of the surroundings and $\alpha = 15$ W/K. If the refrigerator has an efficiency of 30%, what is its power consumption on a day when $T_S = 26°C$?

$0.3P = 15(26-6)$
$P = 1$ kW

**A** 1 kW　　**B** 2 kW
**C** 3 kW　　**D** 4 kW　　[1]

14. A lunar eclipse can only occur when the moon's phase is

**A** New moon.　　**B** Full moon.
**C** First quarter.　　**D** Last quarter.　　[1]

15. Two stars in the night sky are observed to have the same apparent brightness, but one is known to be at a distance of 10 light years and the other at a distance of 20 light years. What is the ratio of the total power radiated by the more distant star to that radiated by the nearer star?

power $\propto$ brightness

inverse square law

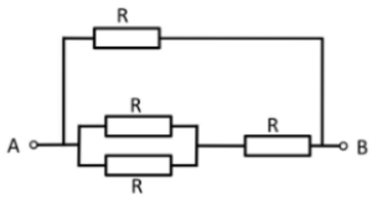
**A** 1.0　　**B** 2.0
**C** 3.0　　**D** 4.0　　[1]

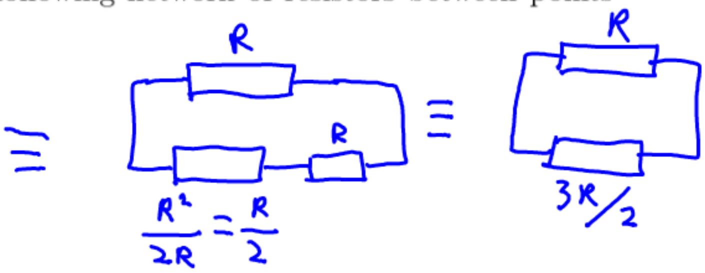
16. An exoplanet is observed to orbit a nearby star at a distance of 0.4 A.U. with a period of 3 days. If a second exoplanet is observed to orbit the same star with a period of 24 days, what must be its orbital radius?
(N.B. 1 A.U. is an *Astronomical Unit* and is the distance the Earth orbits the Sun (1 A.U. $= 1.49 \times 10^8$ km)).

**A** 1.6 A.U.　　**B** 3.2 A.U.
**C** 1 A.U.　　**D** 16 A.U.　　[1]

Kepler's 3rd Law, $T^2 \propto r^3$

17. What is the resistance of the following network of resistors between points A and B?




A R/2 B 5R/3
C R D 3R/5 [1]

$R_T = \frac{3R^2/2}{5R/2}$

18. The primary coil of a transformer is connected to an alternating voltage supply of 240 V and draws a current of 1 A. The secondary coil is connected to a resistor and delivers a voltage of 120 V. If there are 50 turns in the primary coil, how many turns are there in the secondary coil?

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

A 240 B 100
C 25 D 50 [1]

19. An electric motor is driven by a battery of voltage 6 V and draws a current of 1 A. If the motor is used to lift vertically a block of mass 100 g, what is the vertical velocity of the mass?

$P = Fv = IV$

$v = \frac{6 \times 1}{0.1 \times 10}$

A 12 m/s B 6 m/s
C 10 m/s D 0.6 m/s [1]

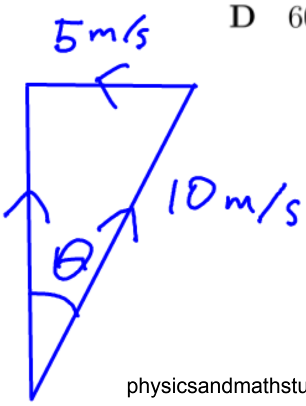
20. A toy car of mass 10 g rests on a slope of inclination 30°. Neglecting friction, What is its acceleration down the slope?

$a = g \sin 30$

A 10 m/s$^2$ B 2.5 m/s$^2$
C 8.7 m/s$^2$ D 5.0 m/s$^2$ [1]

21. A boat crosses a river of width 100 m and flowing in the east-west direction. The water in the river flows from east to west at a speed of 5 m/s. The boat can travel at a speed of 10 m/s. The boat leaves one bank and the skipper wants to reach the point directly on the opposite bank. What course must she steer?

A 30°W B 20°W
C 30°E D 60°E [1]




$\sin\theta = \frac{5}{10}$

$\theta = 30°$

## Written answers (20 marks)

22. A catapult consists of a massless cup attached to a massless spring of length $l$ and of spring constant $k$. If a ball of mass $m$ is loaded into the cup and the catapult pulled back to extend the spring to a total length $x$, what velocity does the ball reach when launched horizontally? [2]

The catapult is then used to launch the ball vertically. If the spring is extended to the same total length of $x$ before release, to what velocity does the catapult now accelerate the ball? [2]

What height above its position at launch will the ball reach if launched vertically? [2]

i) $KE = EPE$

$$\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$$

$$v = \sqrt{\frac{k}{m}}(x-l)$$

ii) $$\frac{1}{2}mv_1^2 + mg(x-l) = \frac{1}{2}k(x-l)^2$$

$$v_1^2 = \frac{k}{m}(x-l)^2 - 2g(x-l)$$

$$v_1 = \sqrt{\frac{k}{m}(x-l)^2 - 2g(x-l)}$$

iii) $$\frac{1}{2}mv_1^2 = mgh$$

$$h = \frac{k}{2gm}(x-l)^2 - (x-l)$$

23. An electron, initially at rest, is accelerated by a potential V in a vacuum and then travels horizontally in a region of space where there is an electric field, E, and a magnetic field, B. The fields are aligned such that the electron is subjected to a force $e$E upwards and a force $ev$B downwards, where $e$ is the charge of the electron and $v$ is its velocity. If E = 1000 V/m and B = $1 \times 10^{-5}$ T, what is the value of the initial accelerating voltage V for the electron to continue flying undeflected?
(Take $m \approx 10^{-30}$ kg and $e \approx 1.6 \times 10^{-19}$ C) [4]

$$eE \doteq evB$$

$$v = \frac{E}{B} = \frac{1000}{1 \times 10^{-5}} = 1 \times 10^{8}\ \mathrm{ms^{-1}}$$

$$E \approx \frac{1}{2} mu^2 = eV$$

$$V = \frac{\frac{1}{2} \times 10^{-30} \times (1 \times 10^{8})^2}{1.6 \times 10^{-19}}$$

$$= \frac{1 \times 10^{-14}}{3.2 \times 10^{-19}} = \frac{1}{3.2} \times 10^{5}$$

$$= 3.1 \times 10^{4}\ V$$

24. A radioactive source emits a parallel beam of alpha, beta and gamma radiation and is placed 10 cm from a detector, which is sensitive to all forms of radiation and receives 100 counts/sec.

i When a sheet of aluminium of thickness 1 cm is placed in front of the detector, the radiation level is seen to fall to 50 counts/sec.

ii When the source is taken away completely, the radiation levels are seen to fall to 10 counts/sec.

iii When the source is placed 1 cm from the detector, the radiation levels are seen to increase to 400 counts/sec.

In what proportion is the source emitting alpha:beta:gamma particles? [4]

i $\Rightarrow$ $\beta = 50$

ii $\Rightarrow$ background $= 10$

iii $\Rightarrow$ $\alpha = 300$

$\therefore \gamma = 100 - (50 + 10) = 40$

$\Rightarrow \alpha : \beta : \gamma = 300 : 50 : 40$

$= 30 : 5 : 4$

25. A packing company supplies storage boxes in three different sizes: small, medium, and large. All three types of box have the same ratio of width:length and height:length. It is noted that:

A. Eight small boxes fit neatly inside one medium box.

B. The length of the small box is the same as the height of the medium box.

C. The base area (i.e. width times length) of a large box is 9 times ~~smaller~~ larger than the base area of the small box.

D. The lengths of all three boxes added together is 2.4 m.

E. The width of the medium box is twice the height of the small box.

What are the lengths of the three different boxes?

What are the ratios of the width:height and width:length of the boxes? [6]

Let $\frac{\text{height}}{\text{width}} = x$, $\frac{\text{length}}{\text{width}} = y$

A: $8 w_s h_s l_s = w_m h_m l_m$ (1)

B: $l_s = h_m$ (2)

C: $9 w_s l_s = w_l l_l$ (3)

D: $l_s + l_m + l_l = 2.4$ (4)

E: $W_m = 2 h_s$ (5)

(1) $\Rightarrow 8 w_s x w_s y w_s = w_m x w_m y w_m$

$8 w_s^3 = w_m^3$

$\boxed{2 w_s = w_m \Rightarrow 2 h_s = h_m, 2 l_s = l_m}$

(3) $\Rightarrow 9 w_s y w_s = w_l y w_l$

$9 w_s^2 = w_l^2$

$\boxed{3 w_s = w_l \Rightarrow 3 h_s = h_l, 3 l_s = l_l}$

(4) $\Rightarrow l_s + 2 l_s + 3 l_s = 2.4$

$l_s = 0.4\text{m}$, $l_m = 0.8\text{m}$, $l_l = 1.2\text{m}$

(5) $\Rightarrow 2 h_s = w_m = 2 w_s$

$\frac{w}{h} = 1$

(2) $\Rightarrow l_s = h_m = 2 h_s$

$\frac{h}{l} = \frac{w}{l} = \frac{1}{2}$

## Long question (20 marks)

26. An archer draws the string of her bow back a distance of 0.6 m and holds it there with a force of 120 N before releasing an arrow of mass 20 g. What is the speed of the arrow when it leaves the bowstring, assuming that all the energy in the bow is imparted to the arrow? [6]

$$EPE = \frac{1}{2}Fx = \frac{1}{2} \times 120 \times 0.6 = 36\,J$$

$$KE = \frac{1}{2}mv^2 = 36\,J$$

$$v = \sqrt{\frac{72}{0.02}}$$

$$= 60\,ms^{-1}$$

In fact, the stored energy of the bow not only accelerates the arrow, but also the arms of the bow and only a fraction $h$ of the original stored energy is imparted to the arrow. If $h = 25/36$, what is the actual speed of the arrow leaving the bow? [2]

$$\frac{1}{2}mv^2 = \frac{25}{36} \times 36$$

$$v = \sqrt{\frac{2 \times 25}{0.02}}$$

$$= 50\,ms^{-1}$$

The archer aims at a target, which is a distance 50 m away. How long will it take for the arrow to reach the target, assuming the arrow does not slow through air friction? [2]

$$t = \frac{\text{distance}}{\text{speed}} = \frac{50}{50} = 1\text{s}$$

To account for the effects of gravity, estimate how far above the centre of the target the archer must aim to ensure that the arrow strikes the middle. [2]

$\downarrow$ +ve $\quad s = ? \quad u = 0 \quad v = x \quad a = 10\,\text{ms}^{-2} \quad t = 1\text{s}$

$$s = ut + \frac{1}{2}at^2$$

$$= \frac{1}{2} \times 10 \times 1^2$$

$$= 5\text{m}$$

If the arrow is brought to rest in a distance of 5 mm, what is the average force of the arrow strike? [4]

$V_{horizontal} = 50 ms^{-1}$ $\quad V_{vertical} = u + at = 0 + 10(1) = 10 ms^{-1}$

$\Rightarrow V^2 = 50^2 + 10^2 = 2600 (ms^{-1})^2$

$$\frac{1}{2} mv^2 = Fx$$

$$F = \frac{\frac{1}{2} \times 0.02 \times 2600}{5 \times 10^{-3}} = \frac{26}{5 \times 10^{-3}}$$

$$= 5200 N$$

If the target has a mass of 5 kg, at what velocity is it thrown back by the arrow strike? [4]

Conservation of momentum

$$0.02(10\sqrt{26}) + 0 = 5.02 v$$

$$v = \frac{0.2\sqrt{26}}{5.02}$$

$$= 0.2 ms^{-1}$$