AQA
Realising potential

TOPIC TEST

A-level
FURTHER MATHS

Centres of Mass 1

Version 1.0

Specification content coverage: ME1, ME2, ME3, ME4

In this test you will be assessed on:

- The centre of mass of a system of particles
- The centre of mass of a composite uniform body
- The use of integration to find the centre of mass of a uniform lamina
- The use of integration to find the centre of mass of a body formed from rotation about the x–axis
- Application of the above to suspension problems

The test comprises two sections. The questions in section A will test you on the basics of the topic. Those in section B require a bit more thinking.

Section A: The basics

1 *AB* is a uniform rod of length 2 metres.

Explain what is meant by 'uniform'.

[1 mark]

2 A system consists of a light rod, *PQ*, of length 1.5 metres with a mass of 2 kg attached at *P* and a mass of 3 kg attached at *Q*.

Find the distance of the centre of mass of the system from *P*.

[2 marks]

3 Three particles, *P*, *Q* and *R*, are placed in the x-y plane.

Particle *P* has mass m and has coordinates (1, 0)

Particle *Q* has mass $2m$ and has coordinates (2, –1)

Particle *R* has mass $5m$ and has coordinates (–5, 6)

Find the coordinates of the centre of mass of the three particles.

[3 marks]

4 *ABCD* is a uniform rectangular plate.

The length *AB* is L metres and the length *AD* is 0.4 metres.

The plate is freely suspended from the corner *A* such that *AD* makes an angle of 63.5° with the downward vertical through *A*.

Find L giving your answer correct to two significant figures

[3 marks]

5 (a) The area bounded by the curve $y = 9 - x^2$ and the x-axis represents a uniform lamina.

Explain why the centre of mass of the lamina lies on the y-axis.

[1 mark]

5 (b) Find the y-coordinate of the centre of mass of the lamina.

[3 marks]

Section B: A bit more thinking

6 A system consists of three particles placed in the x-y plane.

The particles have masses 1 kg, 2 kg and p kg are they placed at the points with position vectors 2**i** + **j**, 3**i** – 2**j** and a**i** respectively.

The position vector of the centre of mass of this system is 4**i** – 0.5**j**

Find the value of p and the value of a.

[4 marks]

7 (a) A large shop sign is in the shape of a pentagon *ABCDE*. The pentagon is formed from a rectangle *ABDE* joined to an isosceles triangle *BCD*. The pentagon can be modelled as a uniform lamina.

Given that *AB* = 5 m, AE = *12* m and *BC* = *CD* = 10 m, find the distance of the centre of mass of the lamina from *AE*.

[4 marks]

7 (b) The lamina is suspended from the corner *B*.

When a particle of mass 20 kg is attached to the point *C* the line *BD* is vertical.

Determine the density of the lamina, correct to two significant figures.

[2 marks]

8 (a) Show that the centre of mass of a uniform solid hemisphere of radius r is at a distance of $\frac{3r}{8}$ from its plane face.

[5 marks]

8 (b) A child's toy is formed from joining the circular face of a solid hemisphere to the circular face of a solid cone.

The hemisphere has radius r.

The cone has a radius r and height r.

Find the distance of the centre of mass from the common circular face

[4 marks]