
A-level FURTHER MATHS

Circular Motion

Version 1.0

Specification content coverage: MD4, MD5, MD6

In this test you will be assessed on:

- use of position, velocity and acceleration vectors in horizontal circular motion
- the conical pendulum
- appropriate contextual questions involving horizontal circular motion testing understanding of assumptions
- circular motion in a vertical plane
- resolving forces towards the centre of a vertical circle
- understanding of when an object completes a vertical circle

The test comprises two sections. The questions in section A will test you on the basics of the topic. Those in section B require a bit more thinking.

Section A: The basics

- 1 John is skating around an ice rink with a constant speed of 10 m s^{-1} .
He moves in a circle of radius 5 m.
Find the magnitude of his acceleration.
[1 mark]
- 2 The Earth has a radius of 6400 km and can be modelled as rotating with an angular speed of 7.3×10^{-5} radians per second.
Find the magnitude of the acceleration of a person standing on the Equator
[1 mark]
- 3 Elizabeth runs around a circular path on horizontal ground in a park.
Her motion can be modelled by $\mathbf{r} = 12\cos\frac{\pi}{6}t\mathbf{i} + 12\sin\frac{\pi}{6}t\mathbf{j}$ metres
Find the exact magnitude of her acceleration.
[4 marks]
- 4 A coin is on a rough horizontal turntable at a distance of 0.4 m from the centre.
The turntable rotates at a constant angular speed.
The coefficient of friction between the coin and the turntable is 0.2.
Given that the coin does not slip, and assuming $g = 9.8 \text{ m s}^{-2}$, find the maximum angular speed of the turntable.
[4 marks]
- 5 A smooth circular wire of radius r and centre O is fixed in a vertical plane.
A bead of mass 0.3 kg is threaded on the wire.
The bead is set in motion with speed u from the lowest point on the circular wire.
Given that the bead makes a complete vertical circle, show that
$$u > 2\sqrt{rg}$$
[4 marks]

Section B: A bit more thinking

- 6** A pilot, of mass 70 kg, is performing an aircraft manoeuvre at an air show.
She flies the aircraft in a vertical circle of radius 350 m.
At the top of the circle the aircraft is flying upside down with a speed of 35 m s^{-1} .
Assuming that $g = 9.81 \text{ m s}^{-2}$, find the magnitude of the reaction between the pilot and the aircraft at the top of the circle.
- [4 marks]**
- 7 (a)** A particle, P , of mass 4 kg is attached to one end of a light inextensible string.
The string passes through a small, fixed smooth ring O .
A second particle, Q , of mass 6 kg is attached to the other end of the string.
 Q hangs at rest vertically below O .
 P moves with speed 5 m s^{-1} in a horizontal circle of radius r .
Find the acute angle between OP and the vertical.
- [4 marks]**
- 7 (b)** Find r .
- [3 marks]**
- 8 (a)** Martin, who has mass 70 kg, is sitting on the seat of a swing.
The swing is made from ropes which are 2 m in length.
Jo-Jo holds the seat so that the ropes are at an angle of 30° to the vertical and then releases it.
Model Martin as a particle and the ropes as light and inextensible and assume $g = 9.8 \text{ m s}^{-2}$.
Find Martin's maximum speed in the subsequent motion.
- [5 marks]**
- 8 (b)** Explain, giving a reason, whether your answer in **(a)** is likely to be an underestimate or an over estimate of Martin's actual maximum speed.
- [2 marks]**