A-level
FURTHER MATHS

Centre of Mass 1

Mark scheme v1.0

Specification content coverage: ME1, ME2, ME3, ME4

Question	Solutions	Mark
1	Uniform means that the mass acts at the centre of the rod, 1 metre from *A*	1
	Total	**1**
2	$3(1.5) = 5\bar{x}$ $\bar{x} = 0.9$ metres	1 1
	Total	**2**
3	$m\begin{bmatrix}1\\0\end{bmatrix} + 2m\begin{bmatrix}2\\-1\end{bmatrix} + 5m\begin{bmatrix}-5\\6\end{bmatrix} = 8mv$ $\begin{bmatrix}-20\\28\end{bmatrix} = 8v$ $\begin{bmatrix}-2.5\\3.5\end{bmatrix} = v$ Coords = (–2.5, 3.5)	1 Forming equation 1 Totalling LHS 1
	Total	**3**
4	$\tan 63.4^\circ = \frac{0.5L}{0.2}$ L = 0.80 m	1 for use of tan 1 for correct fraction 1
	Total	**3**
5 (a)	The y-axis is a line of symmetry	1 Must use ‘symmetry’
	Total	**1**
5 (b)	$\int_{-3}^{3}(9 - x^2)\,dx = 36$ $\frac{1}{2}\int_{-3}^{3}(9 - x^2)^2\,dx = 129.6$ $129.6 \div 36 = 3.6$	1 1 1
	Total	**3**

6	$1\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}3\\-2\end{bmatrix}+p\begin{bmatrix}a\\0\end{bmatrix}=\left(1+2+p\begin{bmatrix}4\\-0.5\end{bmatrix}\right)$ Using y component $-3 = -0.5\ (3 + p)$ $p = 3$ Using x component $8 + pa = 4\ (3 + p)$ $8 + 3a = 24$ $a=\frac{16}{3}$	1 using x component 1 finding p 1 Using y component and substituting their p 1 finding a
	Total	**4**
7(a)	$60(2.5)+48\ (5+\frac{8}{3})=108\ \bar{x}$ $\bar{x}=\frac{518}{108}=\frac{259}{54}$	1 Use of $\frac{8}{3}$ 1 Forming equation 1 Any correct pairing 1
	Total	**4**
7(b)	8(20)g = 518 ρg ρ = 0.31 kg ml^{-2}	1 where ρ = density 1
	Total	**2**
8 (a)	Volume of hemisphere = $\frac{2\pi r^3}{3}$ $\pi\int_0^r xy^2\mathrm{d}x=\pi\int_0^r x\left(r^2-x^2\right)\mathrm{d}x$ $=\pi\left[\frac{r^2x^2}{2}-\frac{x^4}{4}\right]_0^r$ $=\frac{nr^4}{4}$ $\bar{x}=\frac{\frac{\pi r4}{4}}{\frac{2\pi r^3}{3}}=\frac{3r}{8}$	1 stated or implied by use 1 Use of formula 1 Integrating 1 Substituting correct limits 1
	Total	**5**

8 (b)	Using distances from top of hemisphere $\left(\frac{2}{3}\pi r^3\right)\left(\frac{5r}{8}\right)+\left(\frac{1}{3}\pi r^2\right)(2r)\left(\frac{3r}{2}\right)=\bar{x}\left(\frac{2}{3}\pi r^3\right)$ $\bar{x}=\frac{17r}{16}$ Distance $=\frac{r}{16}$ from plane face	1 Forming equation 1 Volumes correct 1 Distances correct 1
	Total	**4**
	TOTAL	**32**