A-level FURTHER MATHS

Circular Motion

Mark scheme v1.0

Specification content coverage: MD4 MD5 MD6

Question	Solutions	Mark
1	$a = \frac{v^2}{r}$ $a = \frac{10^2}{5}$ = 20 m s^{-2}	1
	Total	**1**
2	$a = r\omega^2$ $a = 6\,400\,000 \times (7.3 \times 10^{-5})^2$ $a = 3.41 \times 10^{-2}$ m s^{-2}	1
	Total	**1**
3	$\mathbf{r} = 12\cos\frac{\pi}{6}t\mathbf{i} + 12\sin\frac{\pi}{6}t\mathbf{j}$ $\mathbf{v} = -2\pi\sin\frac{\pi}{6}t\mathbf{i} + 2\pi\cos\frac{\pi}{6}t\mathbf{j}$ $\mathbf{a} = -\frac{2\pi^2}{3}\cos\frac{\pi}{6}t\mathbf{i} - \frac{2\pi^2}{3}\sin\frac{\pi}{6}t\mathbf{j}$ Magnitude of acceleration = $\frac{2\pi^2}{3}$ m s^{-1}	1 Differentiating twice 1 Obtaining **v** 1 Obtaining **a** 1
	Total	**4**
4	$F = mr\omega^2 = 0.4m\omega^2$ $F \le \mu R$ gives $F \le 0.2mg$ $0.4m\omega^2 \le 0.2mg$ $\omega \le 2.2$ rads^{-1} Maximum = 2.2 rads^{-1}	1 Force towards centre 1 Law of friction 1 Forming inequality 1 Maximum obtained
	Total	**4**

5	KE at bottom of circle $= \frac{1}{2}mv^2 = \frac{1}{2}(0.3)v^2$ PE at top of circle $= mgh = (0.3)\text{g}(2r)$ For complete circles KE at bottom must exceed PE at top $\frac{1}{2}(0.3)v^2 > (0.3)\text{g}(2r)$ $v^2 > 4\text{g}r$ $v > 2\sqrt{r\text{g}}$	1 KE 1 PE 1 Forming inequality 1 Obtains result – must see previous line
	Total	**4**
6	Force towards centre = $\frac{mv^2}{r}$ Force towards centre = $\frac{70(35)^2}{350}$ = 245 N Newton's Second Law $70\text{g} - R = 245$ R = 442 N to 3sf	1 Use of force towards centre 1 value seen or implied 1 Forming equation 1 Correct positive R
	Total	**4**
7 (a)	For particle *Q*, resolve vertically $T = 6\text{g} = 58.8$ N For particle *P*, resolve vertically $T\cos\theta = 4\text{g}$ Hence $6\text{g}\cos\theta = 4\text{g}$ $\theta = \cos^{-1}\left(\frac{2}{3}\right) = 48^\circ$	1 1 1 1
	Total	**4**
7(b)	Force towards centre $= \frac{4(5)^2}{r} = \frac{100}{r}$ Newton's second law horizontally $T\sin\theta = \frac{100}{r}$ $r = \frac{100}{58.8\sin 48}$ = 2.3 m	1 Seen or implied 1 Forming equation 1 Correct r
	Total	**3**

8 (a)	Maximum speed occurs at lowest point of motion KE at lowest point $= \frac{1}{2}mv^2 = 35mv^2$ PE at start when being held = mgh = $70g(2 - 2\cos 30^\circ)$ Conservation of energy $35v^2$ = $70g(2 - 2\cos 30^\circ)$ v = 2.3 m s^{-1}	1 Clearly stated 1 KE 1 PE Forming equation 1
	Total	**5**
8 (b)	Speed will be less so it is an over estimate As Martin has size his centre of mass will make the radius of the circle less, hence the potential energy will be lower and subsequently speed will be as well.	1 1
	Total	**2**
	TOTAL	**32**