

A-level FURTHER MATHS

Centres of Mass 2
Mark scheme v1.0

Specification content coverage: ME5, ME6

Question	Solutions	Mark
1	<p>Resolver vertically $R = W$</p> <p>Law of friction $F = \mu R = \mu W$</p> <p>Slides first so $P > F$ hence $P > \mu W$</p>	<p>1</p> <p>1</p>
	Total	2
2	<p>Position of centre of mass from base of cone</p> $= \frac{h}{4} = \frac{8a}{4} = 2a$ $\tan \theta = \frac{a}{2a} = \frac{1}{2}$ $\theta = 26.6^\circ$	<p>1</p> <p>1 Use of tan to form ratio</p> <p>1</p>
	Total	3
3	<p>Moments about O where anticlockwise is positive</p> $= 4(2) - 5(1) + 2(2) - 1(3) + 3(1) - 2(1)$ $= 5 \text{ Nm}$ <p>As this is positive sense is anticlockwise</p>	<p>1 any three pairs correct</p> <p>1 all six pairings correct</p> <p>1 magnitude correct</p> <p>1 signs consistent to deduce anticlockwise</p>
	Total	4
4(a)	$4\mathbf{i} + \mathbf{j} + -\mathbf{i} - 2\mathbf{k} + \mathbf{F} = \mathbf{0}$ $\mathbf{F} = -3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$	1
	Total	1
4 (b)	$\begin{vmatrix} \mathbf{i} & 1 & 4 \\ \mathbf{j} & 5 & 1 \\ \mathbf{k} & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & 3 & -1 \\ \mathbf{j} & 0 & 0 \\ \mathbf{k} & -1 & -2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & 0 & -3 \\ \mathbf{j} & -4 & -1 \\ \mathbf{k} & 2 & 2 \end{vmatrix} = \begin{bmatrix} -6 \\ 1 \\ -31 \end{bmatrix}$ <p>Magnitude = $\sqrt{998}$</p>	<p>1 use of $\mathbf{r} \times \mathbf{F}$</p> <p>1 any 1 component correct</p> <p>1 all components correct</p> <p>1 use of Pythagoras to find magnitude</p>
	Total	4

5 (a)	Use areas of rectangles as proportional to mass Let \bar{x} be distance of centre of mass from AB $0.6(0.75) + 0.3(3) = 0.9\bar{x}$ $\bar{x} = 1.5 \text{ m}$ $\bar{x} = BC$ as required	1 any area/distance pairing 1 Fully correct equation 1 obtains 1.5 m 1 compares to BC or AD
	Total	4
5(b)	Normal reaction at wall = R Moments about point of contact with floor $R(4.5) = 120(3\cos 60^\circ)$ $R = 40\sqrt{3}$	1 LHS 1 RHS 1
	Total	3
5(c)	Normal reaction at floor = N Resolve vertically $N = 120$ Resolve horizontally $F = R = 40\sqrt{3}$ Law of friction $F \leq \mu R$ gives $F\mu \geq \frac{F}{R} = \frac{40\sqrt{3}}{120} = \frac{\sqrt{3}}{3}$	1 1 1
	Total	3
6(a)	Use volumes as proportional top masses $\pi r^2(4r)(2r) + \frac{1}{3}\pi r^2\left(4r + \frac{kr}{4}\right) = \left(\pi r^2(4r) + \frac{1}{3}\pi r^2(kr)\right)\bar{x}$ $\bar{x} = \frac{96r + r(16 + k)}{48 + 4k}$ $\bar{x} = \frac{112 + k}{48 + 4k}r$	1 for cylinder term on LHS 1 for cone term on LHS 1 for RHS 1 equation formed and rearranged 1
	Total	5
6(b)	$\tan \theta = \frac{\text{radius}}{\bar{x}} = \frac{48 + 4k}{112 + k}$ $\frac{48 + 4k}{112 + k} = \frac{2}{3}$ $k = 8$	1 Use of ratio 1 forming equation 1
	Total	3
	TOTAL	32