AS and A-level FURTHER MATHS

Work done by variable force, EPE and power

Mark scheme

Specification content coverage: MC5, MC6, MC7

Question	Solutions	Mark
1	4.5 J	1
	Total	**1**
2	Work done $= \int_1^3 3x(5-x)\,\mathrm{d}x$ $= \int_1^3 15x - 3x^2\,\mathrm{d}x$ $= \left[\frac{15x^2}{2} - x^3\right]_1^3$ $\left(= \frac{81}{2} - \frac{13}{2}\right) = 34$ J	 1 1 1
	Total	**3**
3	Max speed ⇒ resultant force = 0 $D = 35v \quad \Rightarrow \quad D = 35 \times 58 = 2030$ N $\left(P = Dv \quad \Rightarrow \quad P_{\max} = 2030 \times 58 = 117\,740 \text{ W}\right)$ $P_{\max} = 120\,000$ W (2 sf)	1 1
	Total	**2**
4 a	$D = \frac{8400}{12} = 700$ N	1
	Total	**1**
4 b	Resultant force = ma "700" – 480 = $1350a$ $a = 0.163$ m s^{-2}	1 1
	Total	**2**

5 a	$T = kx \quad \Rightarrow \quad 21 = 0.35k \quad \Rightarrow \quad k = 60\left(\text{N m}^{-1}\right)$	1
	Total	**1**
5 b	$\text{EPE} = \frac{kx^2}{2} = \frac{\text{"60"}\left(0.43\right)^2}{2}$ EPE = 5.5 J (2 sf)	1 1
	Total	**2**
6	Extension = 0.6 metres $\left(\text{EPE} = \frac{\lambda x^2}{2} \quad \Rightarrow\right) \quad 27 = \frac{\lambda\left(\text{"0.6"}\right)^2}{2\left(1.4\right)}$ $\lambda = 210\,\text{N}$	1 1 1
	Total	**3**
7	Assume car can be modelled as a particle to allow us to assume all forces act at the same point. $D - 22v = 1300 \times 0.466$ $D = 22v + 605.8$ $32\,400 = Dv$ $32\,400 = \left(22v + 605.8\right)v = 22v^2 + 605.8v$ $0 = 22v^2 + 605.8v - 32\,400$ $v = 27.0 \quad \text{or} \quad v = -54.5$ $v > 0$ so $v = 27.0\,\text{m s}^{-1}$	1 1 1 1 1 1
	Total	**6**

8 (a)	At start, EPE $\left(= \frac{\lambda x^2}{2l} = \frac{2400(0.8)^2}{2(1.2)} \right) = 640$ J At height h, KE $= \frac{1}{2}(18)v^2 = 9v^2$ GPE $= mgh = 18(9.8)h = 176.4h$ At this height, the extension of the string will be $h - 3.2$ EPE $= \frac{2400(h-3.2)^2}{2(1.2)}$ $= 1000h^2 - 6400h + 10240$ $640 = 9v^2 + 176.4h + 1000h^2 - 6400h + 10\,240$ $9v^2 = -9600 + 6223.6h - 1000h^2$	1 1 (KE and GPE) 1 (EPE in terms of $h - 3.2$) 1 Correct expansion 1 Forming eq (using four energy terms) 1 No errors
	Total	**6**
8 (b)	$v = 0$ at maximum height $0 = -9600 + 6223.6h - 1000h^2$ $h = 3.40$ or $h = 2.82$ but $h > 3.2$ so $h = 3.4$ m (2sf)	1 1
	Total	**2**
8 (c) (i)	If air resistance were considered, work would need to be done to overcome this resistive force. Therefore maximum height would decrease.	1
	Total	**1**
8 (c) (ii)	If string was not light, we would need to consider its potential energy. This would require some of the energy in the system, so maximum height would decrease.	1
	Total	**1**
8 (c) (iii)	If the string is held at the point *O*, then the motion cannot be truly vertical. If it were, it would hit what is holding it.	1
	Total	**1**