AQA
Realising potential

TOPIC TEST

A-level
FURTHER MATHS

Centres of Mass 2

Version 1.0

Specification content coverage: ME5, ME6

In this test you will be assessed on:

- Application of centres of mass as appropriate to problem solving
- Conditions for sliding and toppling of rigid bodies
- Use of the moment of a force as force times distance and **r** x **F**
- Understanding the concept of a couple
- Reducing any system of forces to an equivalent system

The test comprises two sections. The questions in section A will test you on the basics of the topic. Those in section B require a bit more thinking.

Section A: The basics

1 A uniform cube of side $6a$ and weight W rests on a rough horizontal plane surface.

A horizontal force P is applied to the centre of one vertical face.

The coefficient of friction between the cube and the plane surface is μ

The cube remains upright and slides along the plane surface.

Find an inequality for P in terms of W and μ

[2 marks]

2 A uniform solid cone has weight W, height $8a$ and base radius a

The cone rests in equilibrium with its plane face on a rough plane which is inclined at an angle θ to the horizontal.

The angle θ is gradually increased from 0°.

The plane is sufficiently rough to prevent slipping.

Find the value θ when the cone is about to topple. Give your answer to three significant figures.

[3 marks]

3 Forces $5\mathbf{i} + 4\mathbf{j}$, $-\mathbf{i} + 2\mathbf{j}$, and $2\mathbf{i} - 3\mathbf{j}$ act through the points (2, 1), (–1, 3) and (1, –2) respectively. The units are newtons and metres

Show that the forces have a total moment of 5 Nm in an anticlockwise direction about the origin.

[4 marks]

4 (a) Three forces $4\mathbf{i} + \mathbf{j}$, $-\mathbf{i} - 2\mathbf{k}$, and **F** act at the points whose coordinates are (1, 5, 0), (3, 0, –1) and (0, –4, 2) respectively.

Given that the three forces are equivalent to a couple about the origin, find **F**

[1 mark]

4 (b) Find the magnitude of the couple.

[4 marks]

Section B: A bit more thinking

5 (a) An advertising sign consists of two rectangles, *ABCD* and *EFGH*, fixed rigidly together. Each rectangle has the following dimensions:

AB = *DC* = 0.4 m and *BC* = *AD* = 1.5 m

HE = *GF* = 3 m and *GH* = *FE* = 0.1 m

DGHC are in a single line where *HC* = *DG* = 0.15 m

The sign can be modelled as a uniform lamina.

Show that the centre of mass of the sign lies on the line joining *C* to *D*.

[4 marks]

5 (b) The sign is placed with its side *EF* on rough horizontal ground and its side *AB* against a smooth vertical wall.

The sign rests in equilibrium at an angle of 30^{o} to the vertical.

The weight of the sign is 120 Newtons.

By taking moments, show that the magnitude of the normal reaction force between the sign and the wall is $k\sqrt{3}$ Newtons, where k is an integer to be found.

[3 marks]

5 (c) The coefficient of friction between the sign and the ground is μ

Show that $\mu \geq \frac{\sqrt{3}}{3}$

[3 marks]

6 (a) A toy rocket consists of two sections.

The lower section of the rocket may be modelled as a uniform solid cylinder with radius r and height $4r$.

The upper section of the rocket may be modelled as a uniform solid cone of radius r and height kr.

Show that the centre of mass of the toy rocket is at a distance of

$$\left(\frac{112+k}{48+4k}\right)r$$

from the base of the toy rocket.

[5 marks]

6 (b) The toy rocket is placed on a rough plane which is inclined at an angle of $\tan^{-1}\frac{2}{3}$ to the horizontal. Given that the toy rocket is on the point of toppling and does not slide, find the value of k.

[3 marks]