
AS and A-level FURTHER MATHS

Work done by variable force, EPE and power

Specification content coverage: MC5, MC6, MC7

In this test you will be assessed on:

- work done by a variable force.
- Elastic Potential Energy using modulus of elasticity
- using conservation of energy to solve problems
- power.

The test comprises two sections. The questions in section A will test you on the basics of the topic. Those in section B require a bit more thinking.

Section A: The basics

- 1** A block of mass m kg is attached to one end of an elastic string of natural length 60 cm and modulus of elasticity 240 N.
The other end of the string is attached to a fixed point P on a horizontal surface.
The string is then extended so that the block rests on the horizontal surface, 75 cm from P .
Which of the following is the elastic potential energy stored in the string at this point?
- 4.5 J 9 J 112.5 J 450 J
- [1 mark]**
- 2** When a particle P is at distance x metres from the origin, it experiences a force of magnitude $3x(5 - x)$ newtons.
Given that this is the only force acting on P , find the work done by the force in moving P from $x = 1$ to $x = 3$.
- [3 marks]**
- 3** A motorcycle has a maximum speed of 58 m s^{-1} .
When the motorcycle is travelling at a speed of $v \text{ m s}^{-1}$ on a straight horizontal road, it experiences a resistance force that can be modelled as having magnitude $35v$ newtons.
Find the maximum power output of the motorcycle, giving your answer to an appropriate degree of accuracy.
- [2 marks]**
- 4** A car of mass 1350 kg is being driven along a horizontal road against a constant resistive force of 480 N.
The power output of the engine is 8400 W and the car is travelling with speed 12 m s^{-1} .
- (a)** Find the driving force of the engine at this point.
- [1 mark]**
- (b)** Find the car's acceleration at this point.
- [2 marks]**

5 The tension in an elastic spring is 21 N when its extension is 35 cm.

5 (a) Find the value of k , the constant of proportionality between the tension in the spring and its extension.

[1 mark]

5 (b) The spring is then stretched further so that its extension is now 43 cm.
Find the elastic potential energy stored in the spring at this point, giving your answer to an appropriate degree of accuracy.

[2 marks]

6 An elastic spring of natural length 1.4 metres is compressed so that its length is 0.8 metres.

Given that it requires 27 J of work to complete this compression, find the modulus of elasticity of the spring.

[3 marks]

Section B: A bit more thinking

- 7 A car of mass 1300 kg is travelling along a straight horizontal road. The resistance to its motion is proportional to its speed.
- When its power output is 44 550 W, it has a maximum speed of 45 m s⁻¹.
- Find its speed at the instant when its acceleration is 0.466 m s⁻² and the power output of its engine is 32 400 W.
- State clearly any assumptions you have made and how you have used them
- [6 marks]**
- 8 **In this question use $g = 9.8 \text{ m s}^{-2}$.**
- A ball of mass 18 kg is attached to one end of a light, elastic string of natural length 1.2 m and modulus of elasticity 2400 N. The other end is held at a point O.
- The ball is released from rest from a point 2 m vertically below O. You may assume that air resistance can be ignored.
- Let h denote the height of the ball, in metres, above its starting point.
- 8 (a) Show that, when $h > 3.2$, the speed of the ball, $v \text{ m s}^{-1}$, satisfies the equation
- $$9v^2 = -9600 + 6223.6h - 1000h^2$$
- [6 marks]**
- 8 (b) Hence find the maximum height the ball reaches above its starting point.
- [2 marks]**
- 8 (c) (i) Explain, with justification, how your answer to part (b) would change if we considered air resistance.
- [1 mark]**
- 8 (c) (ii) Explain, with justification, how your answer to part (b) would change if we did not assume that the string was light.
- [1 mark]**
- 8 (c) (iii) State one criticism of the model used for the motion of the particle in this question.
- [1 mark]**